São Paulo State University – UNESP
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OPTIMIZATION OF THE PRODUCTION PROCESS IN AN AUTOMOTIVE SPRING INDUSTRY

Bauru
2021

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OTIMIZAÇÃO DO PROCESSO DE PRODUÇÃO EM UMA INDÚSTRIA DE MOLAS AUTOMOTIVAS

Tese apresentada como requisito parcial à obtenção do título de Doutor em Engenharia de Produção pelo Programa de Pós-Graduação em Engenharia de Produção da Faculdade de Engenharia de Bauru, da Universidade Estadual Paulista, UNESP, campus Bauru.

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Bauru

2021

A553o

Andrade, Pedro Rochavetz de Lara
Optimization of the production process in an automotive spring industry / Pedro Rochavetz de Lara Andrade. -- Bauru, 2021
99 f. : tabs., fotos

Tese (doutorado) - Universidade Estadual Paulista (Unesp), Faculdade de Engenharia, Bauru
Orientador: Silvio Alexandre de Araujo
Coorientadora: Adriana Cristina Cherri

1. Corte de Estoque. 2. Dimensionamento de Lotes. 3. Modelagem Matemática. 4. Geração de Colunas. I. Título.

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Aos 25 dias do mês de agosto do ano de 2021, às 14:00 horas, por meio de Videoconferência, realizou-se a defesa de TESE DE DOUTORADO de PEDRO ROCHAVETZ DE LARA ANDRADE, intitulada **OPTIMIZATION OF THE PRODUCTION PROCESS IN AN AUTOMOTIVE SPRING INDUSTRY**. A Comissão Examinadora foi constituida pelos seguintes membros: Prof. Dr. SILVIO ALEXANDRE DE ARAUJO (Orientador(a) - Participação Virtual) do(a) Departamento de Matematica / Câmpus de São José do Rio Preto - UNESP, Profª. Drª. KELLY CRISTINA POLDI (Participação Virtual) do(a) Instituto de Matemática, Estatística e Computação Científica - IMECC / Universidade Estadual de Campinas - UNICAMP, Profª. Drª. SONIA CRISTINA POLTRONIERE SILVA (Participação Virtual) do(a) Departamento de Matemática / Faculdade de Ciências de Bauru - UNESP, Profª. Drª. ALINE APARECIDA DE SOUZA LEÃO (Participação Virtual) do(a) Departamento de Matemática / Universidade Estadual de Londrina (UEL), Prof. Dr. ADEMIR APARECIDO CONSTANTINO (Participação Virtual) do(a) Departamento de Informática / Universidade Estadual de Maringá - UEM. Após a exposição pelo doutorando e arguição pelos membros da Comissão Examinadora que participaram do ato, de forma presencial e/ou virtual, o discente recebeu o conceito final: APROVADO. Nada mais havendo, foi lavrada a presente ata, que após lida e aprovada, foi assinada pelo(a) Presidente(a) da Comissão Examinadora.

Prof. Dr. Silvio Alexandre de Araujo

Pedro Rochavetz de Lara Andrade

Optimization of the production process in an automotive spring industry

This thesis is presented as a partial requirement for obtaining the title of Doctor in Production Engineering by the Post-Graduate Program in Production Engineering at the Bauru College of Engineering, São Paulo State University, UNESP, Bauru campus.

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Bauru
2021

DEDICATION

To my parents, Mayra Viviane Rochavetz de Lara and Mauro Roberto de Andrade, who gave me the opportunities that brought me here.

To my wife, Camilla Schulhan Moreira, for her enormous support, without which this work would not have been possible.

To my son, Gabriel Schulhan Rochavetz, for giving meaning to the achievements.

To my past, my present and my future, with all my love.

ACKNOWLEDGMENTS

All my gratitude to God, for all that has been given me, for my life, my purpose, and now for yet another achievement.

I am deeply grateful to my advisors, Prof. Silvio Alexandre de Araujo, Ph.D. and Prof. Adriana Cristina Cherri, Ph.D., for the countless contributions during all these years of work, without which this thesis would not have been possible. For so many careful revisions, so many valuable meetings and discussions that shaped this work little by little. I am also grateful for their patience at so many times when execution was below the expected pace. Above all, I thank them for sharing their experiences, enriching my training, in addition to the professional examples that I received and will continue to guide my career.

I thank my doctorate colleagues, for exchanging experiences, tips, collaborations and for making this journey lighter and more fun. In particular, my deep gratitude to my colleague Prof. Felipe Kesrouani Lemos, Ph.D., for his valuable help in crucial moments of the work. His contributions substantially influenced the quality of this thesis.

I also thank my mother, Mayra Viviane Rochavetz de Lara, for truly rooting for me, for feeling accomplished through my success, for her love that serves as a support and example for me. To my family and friends, for the company, the encouragement, the friendship and for understanding my many absences in recent years. I am especially grateful to my great friends, Karine Kowertz and Fábio Júlio de Lima, for so many translations, help with the text, and for always being by my side, including in this accomplishment.

All my gratitude and love to my wife, Camilla Schulhan Moreira, for being by my side through all these years. For all the support, understanding, encouragement and for all that was necessary to overcome the enormous challenges that came our way during this doctoral degree.

Finally, I thank my son, Gabriel Schulhan Rochavetz, for transforming my life in such a special way, redefining priorities, bringing extra motivation to complete this course and to accomplish so many other things in life.

ABSTRACT

In this thesis, the Cutting Stock Problem is studied, considering three approaches set in an automotive spring industry. The first two approaches deal with the optimization of the process of one-dimensional bar cutting, minimizing material losses and inventory costs in Integrated Lot Sizing and Cutting Stock Problems. Two mathematical models and their respective solution methods based on column generation were proposed. The first refers to short term decisions, considering parallel machines and matters relative to the items and final products. The second model focuses on the medium term, considering the purchase of objects as one of the decision variables, besides demand, inventory costs and limits for objects, items, and final products. The third approach considers assigning items to the hardening furnace as a Cutting Stock Problem. The proposed mathematical model is based on an arc flow formulation. Results with real data show that the three mathematical models obtained significantly superior solutions in viable computational time, in comparison to the company's practice. Tests with random instances were performed allowing for an analysis of the influence of several parameters of these problems. Generally, small instances with large items present better results, with reduced gaps and computational times.

Keywords: One-dimensional Cutting Stock Problem; Lot Sizing Problem; Mathematical Modeling; Column Generation; Automotive Industry.

RESUMO

Nesta tese, o Problema de Corte de Estoque é estudado, considerando três abordagens ambientadas em uma indústria de molas automotivas. As duas primeiras abordagens tratam da otimização do processo de corte unidimensional de barras, minimizando perdas de material e custos de estoque em Problemas Integrados de Dimensionamento de Lotes e Corte de Estoque. Foram propostos dois modelos matemáticos e respectivos métodos de solução baseados em geração de colunas. O primeiro deles trata de decisões a curto prazo, considerando máquinas paralelas e questões relativas aos itens e produtos finais. O segundo modelo está focado em questões a médio prazo, considerando a compra de objetos como uma das variáveis de decisão, além de demanda, limites e custos de estoque de objetos, itens e produtos finais. A terceira abordagem trata da alocação de itens ao forno de têmpera como um Problema de Corte de Estoque. O modelo matemático proposto baseia-se em uma formulação de fluxo em arcos. Resultados com dados reais mostram que os três modelos matemáticos obtiveram, em tempo computacional viável, soluções significativamente superiores em comparação à prática da empresa. Testes com instâncias aleatórias foram realizados, permitindo uma análise da influência de diversos parâmetros destes problemas. Em geral, instâncias pequenas com itens grandes apresentam melhores resultados, com *gaps* e tempos computacionais reduzidos.

Palavras-chave: Problema de Corte de Estoque Unidimensional; Problema de Dimensionamento de Lotes; Modelagem Matemática; Geração de Colunas; Indústria Automotiva.

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List of Acronyms and Abbreviations

3-ILSCSP	3-Level Integrated Lot Sizing and Cutting Stock Problem
cm	centimeter
CSP	Cutting Stock Problem
ILSCSP	Integrated Lot Sizing and Cutting Stock Problem
kg	kilogram
LSP	Lot Sizing Problem
mm	milimeter
OF	Objective Function
OPL	Optimization Programming Language
RAM	Random Access Memory
s	second

Summary

1. INTRODUCTION

The Brazilian automotive industry emerged around 1900, in the beginning the focus was on the production of trucks and utility vehicles (Torres, 2011). The production of passenger cars only reached significant values in the national scenario in the late 1960s (Torres, 2011). Currently, it is the fourth largest industry in Brazil, representing more than 7% of industrial Gross Domestic Product (National Confederation of Industry of Brazil, 2021). Car production by large multinational companies strengthens the local supply of auto parts and other products, as is the case of the studied company, generating indirect jobs and decreasing production costs (Sesso Filho *et al*. 2004).

Considering the production of automotive springs, there are considerable differences between car springs and truck springs. In general, coil springs are used for car suspension. However, the springs used in the suspension of trucks are composed of overlapping cut steel items to provide cushioning for these truck systems. So, steel bars are cut lengthwise to produce the various items (springs) that make up each type of truck spring bundle. Figure 1 illustrates four types of truck spring bundles.

Figure 1 – Examples of truck spring bundles. Source: *Mecânica, Torno e Solda 3M* (2021).

For companies which production process involves the cutting of larger objects into smaller needed items, minimizing the loss of raw material becomes important, and the Cutting Stock Problem (CSP) arises (Kantorovich, 1960; Gilmore and Gomory, 1961, 1963). This is a classic problem of combinatorial optimization that can be found in a wide range of industries, such as paper, steel, plastic, wood, springs, among others (Abuabara and Morabito, 2009;

Toscano *et al.* 2017). Problems of this kind require a little variety of small items to be fully allocated to a selection of objects of fixed size, which may be identical or heterogeneous (Wäscher *et al.* 2007). According to Hifi (2002), it is well know that a large number of possible cutting patterns in this problem means it is highly complex, making it difficult to reach a good quality solution.

The production process of the spring factory considers the one-dimensional cutting of objects (steel bars) into smaller items (springs), aiming at meeting a demand for objects, items, and final products (spring bundles), which result from the assembly of the items. To acquire the necessary properties, the items that make up each type of spring bundle need heat treatment. One of the processes of this heat treatment is hardening, for which it is necessary to assign the items inside a furnace. The quality of this assignment determines the furnace's productivity, impacting its operating time and energy expenditure.

In this thesis, an automotive spring factory is studied aiming at improving the efficiency of the truck springs sector. Therefore, the production process of this sector is analyzed focusing in the processes of a higher volume of items and higher loss of resources (cutting and hardening). Three approaches are applied, two regarding the process of one-dimensional cutting of steel bars into items (springs), seeking to reduce the losses of steel and stocking costs. One of these approaches refers to matters relative to the company's short term and the other considers medium term decisions. These two approaches, besides CSP, already defined, also consider the Lot Sizing Problem (LSP), in which the minimum cost of production is sought through a balance between setup and inventory costs, while meeting the demand for all items (Wagner and Whitin, 1958; Pochet and Wolsey, 2006). The third approach is about assigning items to the hardening furnace, seeking to maximize the amount of assigned items, minimize furnace setups and thus increase daily production.

An important issue is the criteria used to classify the approach of each of the three studies. These are the criteria proposed by Melega *et al.* (2018). The authors made a literature review, and papers approaching the Integrated Lot Sizing and Cutting Stock Problem (ILSCSP) were classified. In their majority, the studies addressed applications either in the paper industry (Poltroniere *et al.* 2008; Poldi and De Araujo, 2016; Leão *et al.* 2017) or the furniture industry (Gramani *et al.* 2009, 2011; Vanzela *et al.* 2017). Other industries addressed are the textile industry, and the industries of copper, steel, and aluminum.

The degree of integration between both problems has also been considered by Melega *et al.* (2018) when categorizing the selected articles. The two integration factors analyzed during this classification were time and production. The first factor of integration is the connection between the periods of time given by the stock, inside a determined planning horizon. The second factor contemplates three production levels. The first one deals with the purchase or production of objects, as well as their stock levels and, depending on the case, demand. The second production level deals with cutting objects into items, considering production capacity, inventory relations, and demand. The third level of production is about assembling items into final products, the stock levels, and demand for the final products. The authors categorized the studies according to which of these production levels have been considered in the model and whether it contemplates multiple time periods.

This thesis is organized into papers regarding each of the three approaches from Chapters 3, 4 and 5. The Chapter 1 initiates with the introduction. Chapter 2 contains the explanation of the production process, with information pertinent to the three studies. The same happens with Chapter 6, which presents a general conclusion for the thesis as a whole. Regarding Chapters 3, 4 and 5, each one presents its own literature review, mathematical model, computational results, discussion and conclusions.

The paper from Chapter 3 has already been published (Andrade *et al.* 2021), the paper from Chapter 4 is under revision after being submitted to an international journal and the paper from Chapter 5 is close to a submission. To avoid information being repeated throughout this thesis, the chapters here presented are not identical to the papers. Naturally some information has been cut or transferred to other chapters so that this thesis stands as a single file.

The paper in Chapter 3 approaches the one-dimensional CSP applied to the steel bar cutting process of the spring industry. The goal is to reduce storage costs together with the losses. As the focus of the study is on short term issues, parallel machines were considered as well as their operational constraints. However, the focus on the short term makes it difficult to consider issues related to the objects, as it has a long delivery period. The model includes demand, inventory costs and limits for items and final products. Therefore, according to the classification proposed in Melega *et al.* (2018), considers production levels 2 and 3, and multiple time periods (-/L2/L3/M), being called ILSCSP.

In Chapter 4, the one-dimensional CSP is also applied to the process of cutting steel bars with the objective of reducing losses of steel and inventory costs. In this approach, a medium-term horizon is considered, enabling the consideration of object related issues. However, unlike the article in Chapter 3, parallel machines are not addressed. The approach contemplates the purchase of bars and its one-dimensional cutting process to produce the springs, and the assembly of spring bundles, in multiple time periods. Demand, inventory costs and limits are considered in every production level, even for bars (objects-level 1), springs (items-level 2) and spring bundles (final products-level 3). The purchase of bars is one of the decision variables in the model. Therefore, this study takes into account every component of the classification presented in Melega *et al.* (2018), being classificated as an ILSCSP of the category (L1/L2/L3/M).

Lastly, the study presented on Chapter 5 applies the one-dimensional CSP to optimize the assignment of items to the furnace of the hardening process, an approach that has not been found in literature by the authors. To enable the implementation of the model, several operational constraints are considered such as the need for the items to be supported by beams, limits for the bending machines after the furnace and possible formula for each item. The focus of this approach is the short term as only one period is considered, representing one day of production. The proposed mathematical model is based on an arc flow formulation, considering additional elements to the one proposed by Valério de Carvalho (2002). Considering the approach taken by Melega *et al.* (2018), this study considers level 2 of production and only one period of time (-/L2/-/S), being named CSP.

Considering all that has been exposed in this introduction, the author states that the essence of this thesis is in the analysis of the different approaches to the one-dimensional CSP, once this is the central subject of the three articles. The possibilities of these approaches in different contexts, how they relate to each other and to other problems, such as the LSP (Chapters 3 and 4) and the arc flow model (Chapter 5), are also analyzed.

2. DESCRIPTION OF THE PRODUCTION PROCESS

In this chapter, the studied spring factory is described in detail. In section 2.1, the production process of the truck springs sector is explained. In Section 2.2, relevant information to the optimization of the bar cutting process is presented, regarding both the short term and the medium term approaches. In Section 2.3, relevant information to the optimization of the hardening furnace is presented.

2.1 Description of the Production Process

The studied company owns five sectors: truck springs, machining, casting, car springs and pins. In this thesis, the truck springs sector is going to be analyzed, and the processes that compose it are illustrated in Figure 2.

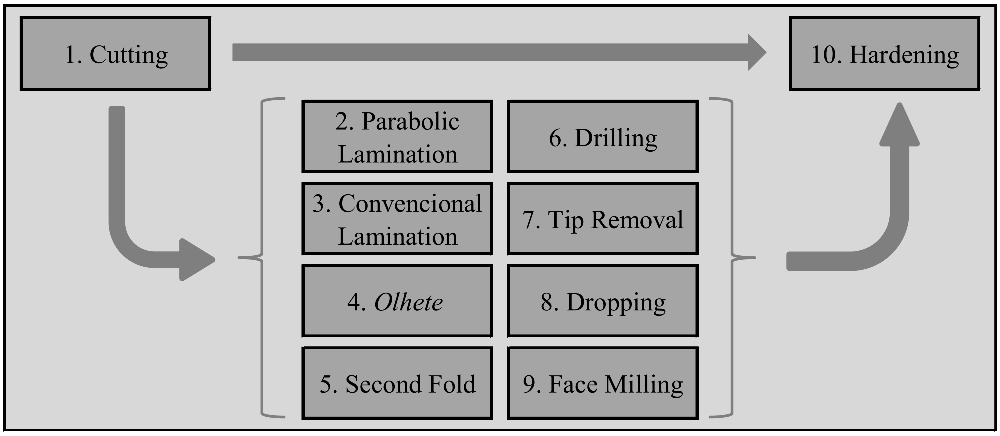

Figure 2 – Processes of the truck springs sector. Source: Author.

As illustrated in Figure 2, all springs must first go through cutting and last through hardening, being that some items only go through these two processes. So, depending on the spring type, needs may vary. It is certain that items that go through Process 3 do not require Process 2, and so is the other way around. The same goes for Processes 4 and 5, which never occur both for the same spring. The need for Processes 6, 7, 8 and 9 varies according to specificities for each spring and there are no precedence rules.

One of the variations among the several types of items is in the need for the bending of the springs. Bending occurs within the Process 10 (Hardening), explained in detail in Section 2.3. The items can be either bent or not bent (straight). Being them bent, they may be of the conventional or parabolic types.

According to the company, the processes in which the highest losses occur are the cutting and the hardening, possibly due to them having the largest volume of items processed. It is understood that both of these processes also have the highest potential for optimization. Because of this, the details of the cutting and the hardening processes are explained in specific sections (Section 2.2 and Section 2.3, respectively). Details of the other processes are explained next, in this section.

2.1.1 Parabolic Lamination

Parabolic lamination refers to the reduction of the thickness and/or the width of parabolic springs (Figure 3). The springs that use this process are 8 mm to 55 mm thick and 50 mm to 120 mm wide. Initially, the part of the material that is going to be laminated goes through an oven until operating temperature is reached. Afterwards, through rolls, the laminator compresses the spring until the specified measure is achieved.

The factory owns two machines for this process, a laminator with automatic handlers and a laminator without automatic handlers. The difference between the two of them is that, in the case of the laminator without automatic handlers, both the control of temperature and exposed time in the oven, as the transport of the material to the cooling counter and its palletization are done by an operator. All of these activities are performed in an automated manner in the case of the laminator with automatic handlers.

Figure 3 – Springs after parabolic lamination. Source: Author.

2.1.2 Conventional Lamination

Conventional lamination seeks to reduce the thickness of the tips of conventional bent springs. The company owns two cells to perform this process, being that the difference between them is in how they achieve the heating of the spring before the lamination itself. One of the cells performs the heating by means of a gas oven and the other through an inductor. After the heating, the operator places the spring in a laminator that utilizes two rolls for compression. Afterwards, an eccentric press is used to perform the tip removal and give the spring its final shape. A set of springs after conventional lamination is pictured on Figure 4.

Figure 4 – Springs after conventional lamination. Source: Author.

2.1.3 *Olhete*

The *olhete* process aims at making a circular fold at the tips of the springs, as illustrated by Figure 5. Initially, the tip of the spring is heated and compressed in a laminator. At this point in the process some springs require a press for bending. Then, a circular movement piston locks the heated tip and performs a fold on the item, which is lastly placed on a pallet.

The studied factory has two cells for the confection of these folds, one counts on hydraulic presses and the other on eccentric presses. The cell with eccentric presses processes springs that need to be bent and the cell with hydraulic presses processes the remaining types of springs.

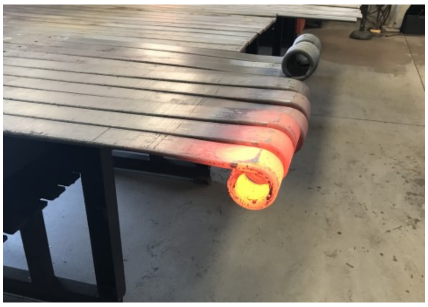

Figure 5 – Springs after olhete. Source: Author.

2.1.4 Second Fold

In this process, the goal is to make circular folds at the tips of the springs. The difference between the second fold and the *olhete* rests on the specification of the fold to be made. While the *olhete* is a closed circular fold at the end of the spring, the second fold makes a more open fold, as seen on Figure 6.

Figure 6 – Springs after second fold. Source: Author.

Moreover, chamfers, semi-folds and folds are also performed at this process according to each item's projects. Chamfers are small cuts made at the tips of the springs (Figure 7). The semi-fold is the making of a curve at the tip of the spring, similar to the second fold but wider (Figure 8). The folds on the spring can be made in various ways, according to necessity (Figure 9).

Figure 7 – Springs after chamfering. Source: Author.

Figure 8 – Springs after semi-fold. Source: Author.

Figure 9 – Springs after fold. Source: Author.

For performing the four activities of the second fold process, the company owns two cells, one being for conventional springs and the other for parabolic springs. For any of the activities, the part of the spring to be processed is first heated in an oven. Following, for chamfering, the spring goes through a laminator which compresses the heated tip, and through an eccentric press which makes the chamfer. The second fold, or semi-fold, is made in a circular movement piston which presses the heated tip and folds it around a stake. The folds are made by pressing the heated part of the spring against a matrix.

2.1.5 Drilling

During the drilling process, holes are drilled into the steel according to each spring's specifications (Figure 10). For this process, the company relies on five machines: an eccentric press with automated belts, three presses for cold drilling and one cell for hot drilling.

All machines operate on manual settings, the eccentric press being the one that performs automatized transportation of springs and adjustments for drilling. This press is capable of processing springs from 70 mm to 101.6 mm wide and 8 mm to 13 mm thick. The eccentric presses for cold drilling are able to process springs from 40 mm to 101.6 mm wide and 5 mm to 15 mm thick. The cell for hot drilling operates with springs from 40 mm to 120 mm wide and thicker than 15 mm.

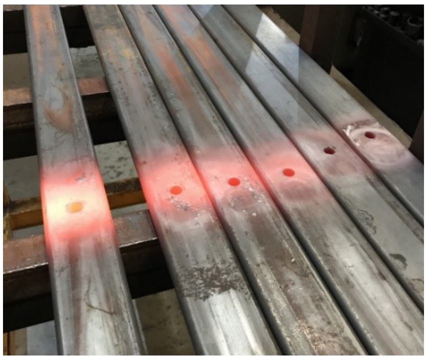

Figure 10 – Springs after drilling. Source: Author.

2.1.6 Tip Removal

The goal of the tip removal is similar to the chamfer: to make cuts at the ends of the spring. The difference is that, as explained on the second fold process, chamfering involves hot cutting and tip removal involves cold cutting. The company owns two eccentric presses for tip removal that may be used on springs between 40 mm and 101.6 mm wide and 5 mm and 15 mm thick. Figure 11 shows a set of springs after this process.

Figure 11 – Springs after tip removal. Source: Author.

2.1.7 Dropping

The dropping process (Figure 12) is similar to the folding. The difference between both is that one is performed on hot material (folding) and the other on cold material (dropping). The company relies on two friction presses to perform the dropping, both with the capacity to process springs from 40 mm to 101.6 mm wide and 5 mm to 15 mm thick.

Figure 12 – Springs after dropping. Source: Author.

2.1.8 Face Milling

The face milling consists of making a small decrease in measures on a specific part of the spring, as shown in Figure 13. Besides improving finishing aspects, this process also allows for higher precision of measure on the part that has been face milled. Precisely due to the need for accuracy, this activity is performed on cold material. The company owns a universal milling cutter for this activity. Initially, the spring is placed and fixed by an operator, who starts the milling cutter so that the indicated part is face milled.

Figure 13 – Springs after face milling. Source: Author.

2.2 Optimization of the Bar Cutting Process

The cutting process consists of cutting objects (steel bars) into specific sized items (springs) which are used in the manufacture of final products (spring bundles). The company owns three machines able to cut up to 4176 items a day altogether: an eccentric press that can produce 1560 items per day and cuts thicknesses up to 20 mm; one metal cut-off grinder disk with a production capacity of 1560 items per day, which cuts up to 35 mm thickness; and an automatic cutting assembly with a production capacity of 1056 items per day, cutting items up to 30 mm thick.

Another operational constraint of these machines is the limit of different types of items that can be used in a given cutting pattern (how the object is cut into smaller items). The automatic machine can cut up to 4 different types of item per cutting pattern, while the other machines accept cutting patterns up to a limit of 3 different types of item. Also, the automatic machine has a limitation in the minimum and maximum length of items that it can produce (from 500 mm up to 2000 mm).

It is worth noticing that there are few adjustments to be made on manual machines when changing the cutting pattern, setup times are short when compared to the cutting time. It occurs in changing the types of items that make up a cutting pattern, and not necessarily in changing the type of bar to be cut. For the automatic machine, there is a short electronic adjustment time for each change of bar type or cutting pattern. But this time can be carried out externally, that is, while the last units of the previous lot are being cut. Based on this discussion, setups will not be considered in the mathematical models that deal with the cutting process, in Chapters 3 and 4.

Cutting patterns of steel bars are designed intending optimal usage whilst producing the demanded springs. Company rules state there must be a minimal usage of 95% of the bar. If not, a new cutting pattern must be formulated. In the case of a loss below 5% for homogenous cutting patterns (with a single type of spring cut from a bar), the company customarily accepts them since cutting patterns are manually designed. Effectively, the company observes average losses within 4.5% and 5% and sells residual steel by weight at an irrelevant price.

In this factory, there is a demand for bars, unit springs and spring bundles. The manufacture of each product is determined by the products' sales history and information regarding current, committed, and minimum and maximum stock. Make-to-Stock is the chosen strategy for production, and the company sets a higher minimum and maximum stock level, the higher the demand for a spring or spring bundle is. In extreme situations of low demand, one seeks the smallest stock possible. The worker responsible for inventory information monitors it daily and later determines how much of each spring and spring bundle shall be produced.

Although there are significant inventories of springs and bars, a reduced inventory of spring bundles is kept by the company, intending to hold springs in stock that may be sold as individual pieces and assembled into spring bundles exclusively when demand presents itself. The company mobilizes some workers of different functions to produce spring bundles, only when it is needed. The size of this working group varies according the demand for spring bundles. Consequently, there is no fixed production capacity for it.

The company is supplied with steel bars standard sized in length, width, and thickness. The time between order and delivery for bars is considered long and might occasionally reach six months. Altogether, 320 varieties of steel bars are acquired, measuring from one to seven meters in length and containing different percentages of carbon, molybdenum, vanadium, niobium, and aluminium.

Instead of being used in the production line, the bars in stock can also be sold to competitors when demand exists, and the price is advantageous. This is only possible because of the company’s high stock levels and the delivery time variation of the steel bars suppliers.

Every bar is solely able to produce springs types that specifically correspond to its characteristics. Hence the division of bars and springs into subgroups of similar features, being that each bar may only belong to one subgroup and produce springs from such subgroup. It is not viable for a bar to produce springs from any other subgroup.

In view of all the characteristics described, it was decided to address this problem in two ways. The first approach (Chapter 3) is focused on short-term issues, considering the company's cutting machines and their specificities. Since object purchases have a long delivery time and, in this case, a period corresponds to one day, object issues were not considered. This is a viable consideration since the high level of inventories of objects kept by the company, means, on a daily horizon, any production decision can be put into practice.

The second approach (Chapter 4) makes a medium-term analysis, in which a period equals two months of production. In this period, it is possible to contemplate bar-related issues. Costs and stock limits, demand, and decisions on the purchase of bars were included in the model. It is worth mentioning that some detailed operational decisions were not considered in this medium-term approach, such as detailed operational aspects of the production machines.

2.3 Optimization of the Hardening Process

The hardening process of the truck spring sector is one of the last processes to be performed on the items before assembling the final products. The process consists in the assignment of items in the hardening furnace. After hardening, the items that need bending go to the benders and then to cooling in oil tanks. The other items go straight to the oil tanks. Finally, the springs pass through the tempering furnace. As the goal of the study in Chapter 5 is to optimize the use of the hardening furnace, the focus of this section is on the hardening furnace itself as well as its relations to the benders.

To carry out this process, the company has one hardening furnace, three benders for conventional items, one bender for parabolic items, four oil tanks for cooling and one tempering furnace. Each day of production, the hardening furnace is turned on for about 9 hours, and energy consumption during this period is among the most significant costs of production.

Due to the high cost of operating the furnace and to avoid production stoppages, the company maintains a large intermediate stock before it goes into the furnace, that is, there are always many items waiting to be hardened. At the end of each day, it is decided which items will be processed the next day. This decision is made prioritizing the items which stock level is closer to the minimum stock, as well as those that have been waiting longer for processing.

In the hardening furnace, the usable width for item assignment is 1,870 millimeters. Therefore, the sum of the length of the assigned springs must be less than or equal to this measurement, the difference between the two values being an empty space in the furnace and, therefore, representing a loss. After assigning items, movable beams transport them along the length of the furnace to be hardened.

Depending on the types of items to be processed, the furnace speed and the temperature must be changed. All types of items are hardened at temperatures between 860°C and 980°C, with possible temperature levels every 10°C. To perform temperature and/or speed change of the furnace, a setup is required. At each setup, to ensure that all item types are processed within the specified conditions, about half the length of the furnace is empty between items of one type and another. Items of one specification pass through the furnace until they reach the end of processing, when items of another specification are already at the beginning of the crossing. At this point, the furnace is stopped until the working temperature of the incoming items is reached. Then, the movable beams perform the steps again, at the speed specified for the springs that are at the beginning of processing.

Theoretically, setups can be performed to either increase or decrease the furnace speed and/or the temperature. However, in practice, the company starts production with light and thin items, which specification requires the coldest furnace (about 860°C to 920°C), in addition to a higher speed (about 20 to 22 minutes to go through the furnace). Throughout the day, items of intermediate thickness and weight are chosen and the thicker and heavier springs are processed at the end of the day at temperatures from around 940ºC to 980ºC, taking around 35 to 41 minutes to go through the furnace. Therefore, the setups currently carried out by the company generate a reduction in speed and an increase in temperature over the course of each day.

In addition, the company defined standardized formulas to simplify the parameterization of furnace speed and temperature. Each formula specifies a temperature and speed to be used as well as the thickness range of the items that can be processed under these conditions. Therefore, each formula can only be used to process items that fall within the defined thickness range but each item can be covered by more than one formula.

The objective of the company is to process at least 30 tons of springs daily. When good assignments are made, empty spaces in the furnace are reduced. Additionally, good production planning allows for fewer setups, which also reduces wasted space in the furnace. High occupancy of the furnace reduces the planned processing time for the day or allows for total production above the forecast. Therefore, the objective of this approach is to maximize the amount of items assigned in the furnace, minimizing the number of setups needed and, consequently, improving furnace productivity.

An important issue to be considered, for the viability of the solution, is the need for all items to be supported by at least two beams, otherwise they will fall, making the assignment unfeasible. Therefore, the distance between the beams must be considered when deciding which items to assign. Figure 14 shows a birds-eye view of the hardening furnace measurements.

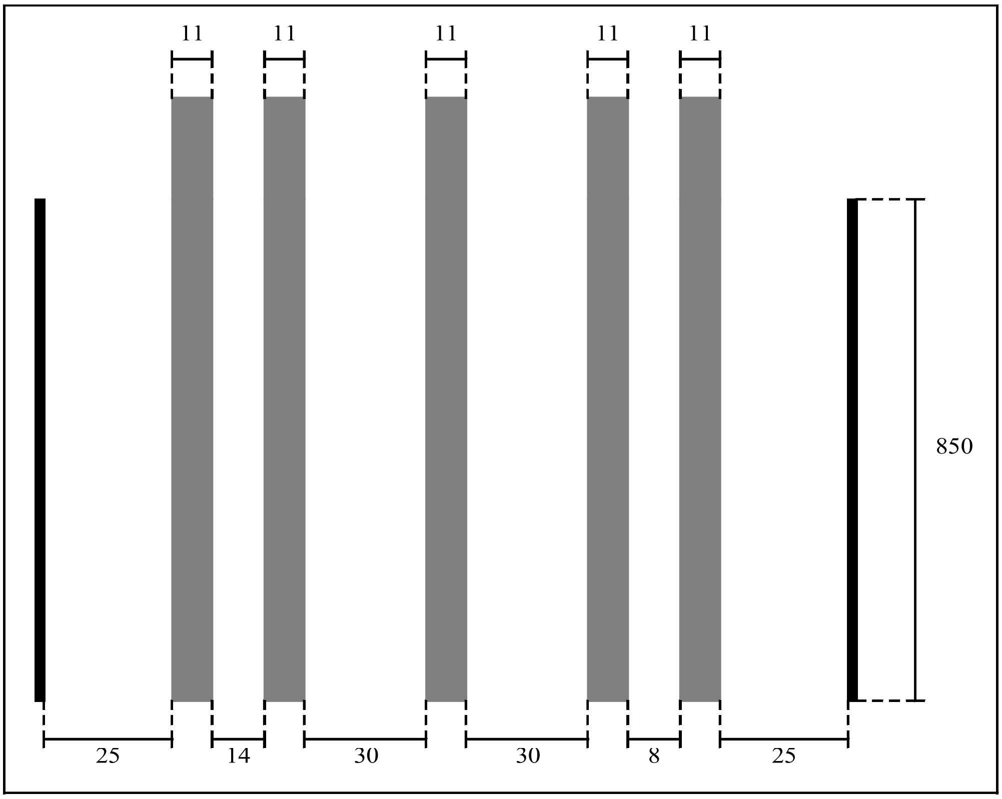

Figure 14 – Internal dimensions, in top view, of the hardening furnace (centimeters). Source: Author.

Each of the gray colored objects in Figure 14 represents a movable beam. At the top of the figure are the measurements of each beam (110 millimeters each), at the bottom the measurements of the spaces between the beams. Each of these measurements is associated with what will be called the furnace section (in the example in Figure 14, there are 11 sections). The sum of all measurements (sections), of the beams and the spaces between the beams, equals the furnace width (1,870 millimeters). On the right side of the figure is the measurement of the length of the furnace (8,500 millimeters). The black lines represent the lateral limits of the furnace and, according to this representation, the items enter the furnace from the top of the figure, are transported by the movable beams to the bottom, where they are released for the next processes. At each step of the beams, the items advance 85 millimeters along the length of the furnace, which in this case is 8,500 millimeters. Therefore, it takes 100 steps for an item to traverse the entire length of the furnace. To change the furnace speed, the time for the completion of each step is changed.

The benders process the items after the hardening furnace. Parabolic bent springs must be processed in parabolic benders, in the same way as conventional bent springs are bent in conventional benders. Items not bent, after hardening in the furnace, go straight to the cooling in the oil tank. Therefore, each bender can process only a subset of items. At each assignment, the number of parabolic and conventional items must respect the number of benders of each type available. Items that are not bent can be freely assigned because they do not use the benders.

The following example illustrates a situation with possible assignments and impossible assignments in a hardening furnace.

2.3.1 Example

In this example, 6 item types are considered, and Figure 15 illustrates their main characteristics. The furnace considered has 5 movable beams. As the purpose of this section is to illustrate possible and impossible assignments, it is not necessary to provide exact measurements for the item types and the furnace. For the assignment of items in the furnace, the following restrictions must be considered: the items must be supported by at least two beams; the limitation on the number of benders must be respected; and for each assignment, only items from the same formula can be assigned together. In this example, consider that the limit of the benders is 2 for conventional items and 1 for parabolic items. As explained in the previous section, straight items do not need to use any bender. Figure 16 shows examples of possible assignments and Figure 17 shows impossible assignments.

Item	Representation	Formula	Type
Item 1	1 - P	1	Parabolic
Item 2	2 - S	2	Straight
Item 3	3 - C	2	Conventional
Item 4	4 - C	1	Conventional
Item 5	5 - S	1	Straight
Item 6	6 - P	2	Parabolic

Figure 15 – Item types and their characteristics. Source: Author.

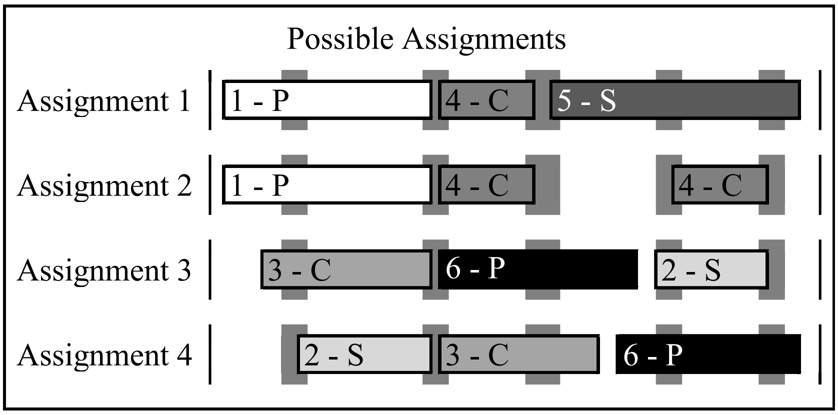

Figure 16 – Illustrative example of possible assignments. Source: Author.

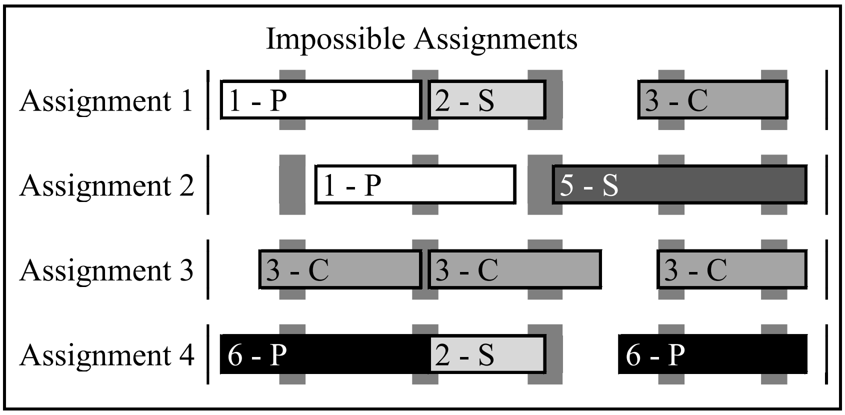

Figure 17 – Illustrative example of impossible assignments. Source: Author.

Note that, in Figure 16, all assignments respect the restrictions. In Figure 17, assignment 1 shows the use of items from different formulas. In assignment 2, item 1 is not supported by two beams and, in assignments 3 and 4, the limits of conventional and parabolic items, respectively, were not respected.

3. THE INTEGRATED LOT SIZING AND CUTTING STOCK PROBLEM IN AN AUTOMOTIVE SPRING FACTORY

In this chapter, the production of truck springs in an automotive spring factory is addressed. The goal is to reduce storage costs together with the losses in the cutting of steel bars. The approach considers the one-dimensional cutting of objects (steel bars) into smaller items (springs) that are assembled into final products (spring bundles). There is demand for items and final products, besides their inventory costs and limits.

Aiming to improve the company's practice, an integrated approach between the CSP and the LSP is proposed. As already demonstrated by several authors (Gramani *et al*. 2009; Vanzela *et al*. 2017; Melega *et al*. 2018; Do Nascimento *et al.* 2020), these two problems are quite dependent, and an integrated approach can improve the global result, compared to treating each problem separately. It is important to state that no optimization approach in the spring industry, similar to this one, has been found in the literature so far.

The main contribution of this chapter lie in the mathematical model proposed to represent the practical problem in an integrated approach. The model was developed to meet the needs of the factory, considering short-term issues. Multiple time periods, parallel machines, stock limits for items and final products are considered in this model, besides machines capacities, and operational constraints such as the limit of item types by cutting patterns, thickness, and length limits in each machine. Besides the cutting process of items (springs), the assembly of final products (bundles) is also considered. According to the future research agenda proposed in the recent review of Melega *et al.* (2018), this chapter helps fill a gap in the literature, by considering multiple, heterogeneous, parallel and capacitated cutting machines, applied to an industrial sector that has not yet been explored in this context.

Regarding the solution method, the simplex method with column generation was used to solve the linear relaxation, according to Gilmore and Gomory (1961), followed by obtaining an integer solution through a computational package. The formulation of the subproblems is new since it considers the specificities of the company; in this case, the limit of item types by cutting pattern, as well as machine limitations and others. The cutting knife limitation, considered by Gilmore and Gomory (1963) in an application in the paper industry, is similar to the consideration of the limit of item types by cutting pattern, performed in this chapter. Note that this is a straightforward solution method with innovative aspects, which is justified because the main focus of this chapter is the application and not the methodology.

The implementation, with extensive computational tests, of such a new approach applied to a factory in the Brazilian automobile sector, among the most important in the country, contributes to both current practice and to the literature. The approach is validated by solving instances with both real and random data. The solution of the spring company instance, obtaining a very significant reduction in losses (almost 50%), demonstrates the quality of the model and the relevance of the study. Solving the random instances allows for analysis of the influence of different parameters, giving a better understanding of the problem and enabling managerial insights that can further improve practical results.

This study has already been published in an international journal (Andrade *et al.* 2021), and the contributions of the author of this thesis were: visits to the company to collect information; conception of the mathematical model; implementation of programming techniques; running the instances; analysis of results; and writing the text.

This chapter is divided into five sections, these being: the literature review (3.1); the proposed mathematical model (3.2); the explanation of the solution method (3.3); the presentation of the computational results (3.4); and the conclusions (3.5).

3.1 Literature Review

Many companies have turned their efforts to integrated approaches, looking for a global optimal solution, naturally better than optimizing isolated problems (Gramani *et al.* 2011). Consequently, studies on the ILSCSP have been increasing (Melega *et al.* 2018). Among the reasons for this, is the great potential for economic gains in several industries, in addition to the recent advances in computation, which allows the approach to address more complex problems.

The ILSCSP basically captures the trade-off between material losses in the cutting process and inventory costs (Poldi and Arenales, 2010). Depending on the approach, inventory costs for items, objects and/or final products are considered. In addition, each practical application results in specific constraints to be added to the model, such as object production, inventory limits, machine capacity, among others.

In a recent literature review of papers that deal with the ILSCSP, Melega *et al*. (2018) found 34 studies conducted over 32 years, nine of which had been published in the last two years of the research (2016 and 2017). Using the same criteria, searching for papers from 2018 onwards, it has been found another six papers.

The present study considers the cutting process of objects, together with the assembly of final products, over multiple periods of time. Therefore, according to the nomenclature proposed in Melega *et al*. (2018), it is an ILSCSP classified as (-/L2/L3/M). In their review, the authors found 9 papers with this same classification. Considering the years of 2018, 2019 and 2020, only the paper of Melega *et al.* (2020) was found with this specific classification.

Considering the papers that approach the same integrated problem (-/L2/L3/M), Ghidini *et al*. (2007) propose a mathematical model for the ILSCSP that arises in a small furniture factory. In order to solving using the Simplex method with column generation, the model is simplified, disregarding the cutting machine setup costs. Two instances with real data are solved, and the authors highlight the importance of considering practical aspects, such as capacity limits and demand fluctuation for the solution's applicability.

A heuristic method, based on Lagrangean relaxation, to solve an ILSCSP in a furniture industry is presented in Gramani *et al*. (2009). The model considers guillotine cutting in two stages, in addition to setup costs for both problems. Shortly after that, in Gramani *et al*. (2011), the model is adjusted, considering item inventory costs and disregarding final product setup. These changes allowed the solution of the model through the Simplex method with column generation.

In Santos *et al*. (2011), the mathematical model of the ILSCSP in a furniture industry is presented. An operational constraint specific to the furniture industry is considered, aiming to reduce the number of saw cycles. For the solution of the model, cutting patterns generated *a priori* are used and the authors suggest that a column generation approach could improve the results.

Suliman (2012) formulate the ILSCSP as a non-linear integer model and propose an algorithm to solve it. The algorithm works backwards, solving the last period first. It is not a complete integration, as it solves the LSP first, and then the CSP. The authors test the method with a practical example in the aluminum industry, and with fictitious instances.

In two sequential papers, Alem and Morabito (2012, 2013), study a small furniture factory and strategies to deal with the risks arising from uncertainty scenarios. In Alem and Morabito (2012), robust models are proposed for the ILSCSP that consider uncertainties in several parameters, such as production and storage costs, and demand. The model is tested with real data and simulated instances. In Alem and Morabito (2013), a deterministic model is presented, which considers, among other constraints, setup costs in the cutting and drilling sector. The authors analyze different models of two-stage stochastic programming to make decisions on risk-neutral or risk-averse strategies.

Vanzela *et al.* (2017) propose a mathematical model to contemplate this integrated approach in a small furniture factory. In order to obtain a practical feasible solution, saw cycles are considered in capacity constraints. The authors compare the integrated model solution with the simulation of factory practice, where decisions are made separately, concluding that the model can improve the production planning of the company.

The work in Wu *et al.* (2017) is essentially theoretical, focusing on the performance of techniques for solving the ILSCSP. The authors propose a new progressive selection algorithm, in addition to two Dantzig-Wolfe decomposition approaches with column generation. The authors claim that the proposed methods are computationally feasible and capable of improving the results in comparison with other methods.

In Melega *et al.* (2020), the authors consider the ILSCSP with sequence-dependent setup times and setup costs, i.e., the cutting patterns must be sequenced in order to obtain a solution for the problem. The solution method uses column generation and the integer problem is solved by decomposition approaches. Computational results, with randomly generated instances, are presented.

Finally, considering papers that approach this integrated problem in a single period (-/L2/L3/S), Farley (1988) study a textile factory, creating two models to describe it, a quadratic and an integer. Both models consider several specificities at the company being considered, such as setup costs in the cutting machines, and minimum and maximum production levels of final products. Lemos *et al.* (2021) study the integration of a one-dimensional CSP in an environment of multiple manufacturing modes, an unexplored approach. The main goal is to adapt the model to solve the practical problem of a concrete pole factory. In addition to solving the real problem, the proposed model is tested with several instances of random data. Table 1 shows the summary of information about each article:

Table 1 – Classification of the related papers from the literature.

Authors	Application	Dimensionality	Periods
Farley (1988)	Textile	Two-dimensional	Single
Ghidini *et al.* (2007)	Furniture	Two-dimensional	Multiple
Gramani *et al.* (2009)	Furniture	Two-dimensional	Multiple
Gramani *et al.* (2011)	Furniture	Two-dimensional	Multiple
Santos *et al.* (2011)	Furniture	Two-dimensional	Multiple
Suliman (2012)	Aluminium	One-dimensional	Multiple
Alem and Morabito (2012)	Furniture	Two-dimensional	Multiple
Alem and Morabito (2013)	Furniture	Two-dimensional	Multiple
Vanzela *et al.* (2017)	Furniture	Two-dimensional	Multiple
Wu *et al.* (2017)	General	One-dimensional	Multiple
Melega *et al.* (2020)	General	One-dimensional	Multiple
Lemos *et al.* (2021)	Construction	One-dimensional	Single

As can be seen from Table 1, most papers deal with the furniture industry in a two-dimensional approach (Ghidini *et al.* 2007; Gramani *et al*. 2009, 2011; Santos *et al.* 2011; Alem and Morabito, 2012, 2013; Vanzela *et al.* 2017). All the studies, except Wu *et al.* (2017) and Melega *et al.* (2020), similar to the present study, have as main objective the solution of a practical application, considering several real word constraints, and also carrying out computational tests with fictitious instances. Futhermore, the Simplex method with column generation is a technique widely used in these approaches (Ghidini *et al.* 2007; Gramani *et al.* 2011; Vanzela *et al.* 2017; Wu *et al.* 2017; Melega *et al.* 2020; Lemos *et al.* 2021).

No mathematical model was found that considers the same aspects that are being considered in this chapter. The most similar is Gramani *et al.* (2011), but the authors only consider the linear relaxation of the model. Moreover, they do not consider parallel machines and operational constraints, which make our subproblems to generate columns totally different from the subproblems considered in Gramani *et al.* (2011).

3.2 Mathematical Model

The proposed mathematical model considers cutting objects (steel bars) into items (springs) and assembling items into final products (bundles). Both items and final products have their own demands, inventory costs and limits. Setups are not considered, as well as the limitation of production capacity of final products, since, as explained in Section 2.2, this does not make sense for the considered company. The limit for item production capacity is considered according to the specification of each cutting machine. Parallel machines are considered, as well as their operational constraints, which will appear in the subproblem, to be explained in section 3.3. Each term used in the mathematical model is defined as follows:

Sets:

I: set of item types $\{i = 1, \dots, |I|\}$ (index *i*);

K: set of object types $\{k = 1, \dots, |K|\}$ (index *k*);

J: set of cutting patterns $\{j = 1, \dots, |N_K|\}$ (index *j*). Where N_k is the number of cutting patterns of object type *k*;

P: set of final product types $\{p = 1, \dots, |P|\}$ (index *p*);

F: set of cutting machines $\{f = 1, \dots, |F|\}$ (index *f*);

T: set of periods $\{t = 1, \dots, |T|\}$ (index *t*).

Parameters:

dr_{it}: demand of item type *i* in period *t*;

dp_{pt}: demand of final product type p in period *t*;

α_{ijk}: quantity of item type *i* produced by cutting pattern *j* from object type *k;*

z_{ip}: quantity of item type *i* in one unit of final product type *p*;

cap_{ft}: production capacity (in number of items) of the machine *f* in period *t*;

c_{jkft}: cost of cutting (mm of bar) of an object type *k* according to the cutting pattern *j* on machine *f* in period *t;*

cr_{it}: storage cost (in mm of bar) of item type *i* in stock at the end of period *t*;

cp_{pt}: storage cost (in mm of bar) of final product type p in stock at the end of period *t*;

$rmin_i$: minimum stock of item type *i*;

$rmax_i$: maximum stock of item type *i*;

$umin_p$: minimum stock of final product type *p*;

$umax_p$: maximum stock of final product type *p*;

L_k: length of object type *k*;

l_i: length of item type i.

Decision Variables:

x_{jkft}: number of objects type k cut according to cutting pattern *j* on machine *f* in period *t*;

y_{pt}: number of final products type p produced in period t;

r_{it}: number of items type i in stock at the end of period t;

u_{pt}: number of final products type p in stock at the end of period t.

The mathematical model is as follows:

$$Min \sum_{t=1}^{|T|}((\sum_{k=1}^{|K|}\sum_{j=1}^{|N_k|}\sum_{f=1}^{|F|} c_{jkft}x_{jkft}) + \sum_{i=1}^{|I|} cr_{it}r_{it} + \sum_{p=1}^{|P|} cp_{pt}u_{pt}) \tag{1}$$

$Subject\ to$:

$$\sum_{k=1}^{|K|}\sum_{j=1}^{|N_k|}\sum_{f=1}^{|F|} \alpha_{ijk}x_{jkft} + r_{i,t-1} = dr_{it} + r_{it} + \sum_{p=1}^{|P|} z_{ip}y_{pt}\ , \qquad \forall\, i,t, \tag{2}$$

$$y_{pt} + u_{p,t-1} = dp_{pt} + u_{pt}, \qquad \forall\, p,t, \tag{3}$$

$$\sum_{i=1}^{|I|}\sum_{k=1}^{|K|}\sum_{j=1}^{|N_k|} \alpha_{ijk}\, x_{jkft} \leq cap_{ft}\ , \qquad \forall\, f,t, \tag{4}$$

$$rmin_i \leq r_{it} \leq rmax_i\ , \qquad \forall\, i,t, \tag{5}$$

$$umin_p \leq u_{pt} \leq umax_p\ , \qquad \forall\, p,t, \tag{6}$$

$$x_{jkft} \in Z_+,\ y_{pt} \in Z_+\ ,\ r_{it} \in R_+\ ,\ u_{pt} \in R_+\ , \qquad \forall\, i,k,j,f,p,t. \tag{7}$$

The cost of cutting an object type k according to cutting pattern j, on machine f, in period t, represented by c_{jkft}, is equivalent to the loss of material, in millimeters, generated by this cutting pattern, that is:

$$c_{jkft} = L_k - \sum_{i=1}^{|I|} \alpha_{ijk}l_i\ , \qquad \forall\, k,j,f,t. \tag{8}$$

In the model (1) - (7), the objective function (OF) (1) minimizes costs with loss of material in all periods (days), as well as the costs of storing items (springs) and final products (spring bundles). Constraints (2) ensure that the demand for all items in all periods is satisfied. Under these restrictions, for all items in all periods, the quantity available in stock at the beginning of the period $(r_{i,t-1})$, plus the quantity produced for this item $(\sum_{k=1}^{K}\sum_{j=1}^{N_k}\sum_{f=1}^{F}\alpha_{ijk}x_{jkft})$ must be equal to the sum of the demand for this item (dr_{it}), the quantity used in the production of all the bundles types $(\sum_{p=1}^{P} z_{ip}y_{pt})$, and the quantity left in stock for the next period (r_{it}). In (3) the demand for final products is satisfied, since for all products p in all periods t, the quantity in stock at the beginning of the period $(u_{p,t-1})$ plus the quantity produced (y_{pt}), should be equal to product demand (dp_{pt}) plus quantity left in stock for next period (u_{pt}). Constraints (4) ensure the production capacity of each cutting machine is respected in all periods. Constraints (5) and (6) establish the minimum and maximum stocks for items and final products, respectively. The initial stocks r_{i0} and u_{p0} are parameters that vary with each instance and can assume any value between the established limits. Finally, in (7), the domains of the decision variables are defined.

3.3 Solution Method

The method used to solve the linear relaxation of the model presented in Section 3.2 is the Simplex method with column generation (Gilmore and Gomory, 1961). A computational package was used to obtain integer solutions. Similar approaches were used by Vanzela *et al.* (2017), Poldi and De Araujo (2016), among others. In this section, the application of this method is explained in detail.

The mathematical model presented in Section 3.2 has a large number of integer variables, x_{jkft} and y_{pt}, making it difficult to obtain an optimal solution. And so, these variables have their integrality constraints relaxed (using x_{jkft} and $y_{pt} \in R_{+}$) and the Simplex method with column generation is used. This model is usually called the "master problem" or “relaxed master problem” when it do not consider integrality constraints for variables x_{jkft} and y_{pt}.

The objective of the method is to add columns (cutting patterns, in this case) to the relaxed master problem in order to improve the OF. Columns are generated through the solution of subproblems, which are mathematical models of the knapsack problem. Iteratively repeating this process until no generated column can improve the OF, the optimal solution of the relaxed master problem is obtained (Poldi, 2007).

Initially, the master problem is solved considering only the homogeneous maximal cutting patterns, that is, cutting patterns containing only one item type as many as possible. This procedure ensures that, even with an initial reduced set of columns, it is possible to find a feasible solution to the master problem (Lemos, 2020). In a feasible solution, each constraint of the master problem is associated with a dual variable, and each column is associated with a reduced cost. The reduced cost of a column is the difference between its coefficient on the OF and the sum of the product of its coefficients on the constraints and the dual values of those constraints. When the reduced cost of a column is negative, it means that the penalty it imposes on the constraints is less significant than the gain it generates in the OF, that is, adding this column to the master problem improves the quality of the current solution.

To generate columns that improve the solution of the master problem, the OF of subproblems, presented in (9), aims to minimize the reduced cost of the generated columns. Considering that the variables x_{jkft}, which represents the cutting patterns, only appears in Constraints (2) and (4), only the dual variables of these two constraints influence the reduced cost value of this variables. Therefore, as can be seen in the OF in (9), only the values of the dual variables referring to Constraints (2) and (4) are used.

At each iteration, the number of subproblems solved is equal to the product of the number of objects (*K*), machines (*F*) and periods (*T*). The items assigned to a cutting pattern are only those that can be produced from a given object type k and by a given machine *f*, ensuring that the cutting patterns generated are technically feasible.

Adaptations were made to the formulation of the subproblems to consider the specificities of this particular company. As a subproblem is generated for each type of object, each machine and each period, resulting in a cutting pattern, the indexes k, j, f and t are fixed in each subproblem. Therefore, the terms L_k, π_{ft} and $imax_f$, given below, are treated as constants. The variable α_{ijk} has the fixed indexes j and k, with only index i varying. In addition to those presented previously, the terms used in the mathematical model of the subproblems are:

Sets:

G_k: Set of item types that can be produced from object type *k*;

H_f: Set of item types that can be produced on machine *f*.

Parameters:

π_{it}: the dual variable of Constraint (2), referring to item type *i* in period *t*;

π_{ft}: the dual variable of Constraint (4), referring to machine *f* in period *t*;

max_f: maximum number of different types of items for a cutting pattern on machine *f*;

M: sufficiently large value.

Variables:

α_{ijk}: quantity of item type *i* produced by cutting pattern *j* from object type *k*;

β_i: whether or not item type *i* is present in the cutting pattern.

The mathematical model of the subproblems (for each $k = 1, \dots, K$, $f = 1, \dots, F$, and $t = 1, \dots, T$) is described below:

$$Min\ L_k - \sum_{i \in G_k \cap H_f} \alpha_{ijk}(l_i + \pi_{it} + \pi_{ft}), \tag{9}$$

$Subject\ to$:

$$\sum_{i \in G_k \cap H_f} \alpha_{ijk} l_i \leq L_k, \tag{10}$$

$$\sum_{i \in G_k \cap H_f} \beta_i \leq max_f, \tag{11}$$

$$\alpha_{ijk} \leq M\beta_i, \quad \forall\ i \in G_k \cap H_f, \tag{12}$$

$$\alpha_{ijk} \in Z_+,\ \ \beta_i = \{0;\ 1\}, \quad \forall\ i \in G_k \cap H_f, \tag{13}$$

In (9)-(13), the objective function (9) minimizes the relative cost of the generated column. Constraints (10) ensure that the sum of the lengths of items in the cutting pattern is not greater than the length of object type *k*. In (11), the number of different item types that can make up a cutting pattern on machine *f* is limited. Furthermore, constraints (12) link the decision variables, ensuring that, if an item type i is not in the cutting pattern (β_i = 0), the variable α_{ijk} for this item is equal to zero. Otherwise, (β_i = 1), the variable α_{ijk} can be any value, provided that the constraints are respected. Finally, (13) defines the domain of variables.

To strengthen the linear relaxation, the value of *M* should be as small as possible. From (10), it is known that $\alpha_{ijk}\ \leq\ L_k/l_i$. Therefore, the value for *M* is the smallest integer number greater than: $L_o/l_p +\ 1$, L_o being the length of the largest object, and l_p the length of the smallest item.

As previously explained, when the relative cost of the column generated by a subproblem is negative, it means that the column improves the quality of the current solution, and it should be added to the master problem. The lower the value, the greater the improvement generated by this column. It is known that adding more columns to the master problem can improve the quality of the final solution, and it also can lead to fewer iterations required to reach this solution. On the other hand, adding more columns increases master problem (relaxed and integer) processing time. Additionally, as it can be seen in Section 3.4, the most significant processing time in the largest instances, especially the real data instance, is required to solve the relaxed master problem, and the integer master problem, rather than the subproblems.

Computational time is also a scarce resource, either because of the risk of memory overflow in very long processing or, in real cases, the need for a solution in the right time for its utilization. Therefore, to reduce the computational time of the master problem and seeking to minimize the impact on the quality of the final solution, a strategy was created. It consists in selecting only better columns to add in the master problem at each iteration, those with the lowest relative costs. After solving $K * F * T$ subproblems in an iteration, the average of the negative relative costs is calculated. Then, only columns generated by subproblems with relative cost less than or equal to the value of the mean multiplied by 0.1 are added to the master problem. This value (0.1) was obtained based on initial tests. Since we are dealing with negative values, the lower the value, the more columns are added.

It is important to clarify that the parameterization of this value to 0.1 causes a small part of the generated columns to be excluded. For example: if the average of negative relative costs is -100. After multiplying by 0.1, columns with relative cost less than -10 would be accepted, that is, the majority of cases would meet the established criteria.

Figure 18 shows a flowchart that summarizes the proposed solution method. In the first iteration, homogeneous maximal cutting patterns are generated and the relaxed master problem is solved. The dual variables are used in the objective function of the subproblems to be solved (K * F * T subproblems at each iteration), the generated columns are added to the master problem according to the explained criterion. It is important to note that the columns considered for calculating the mean are only those that obtain a negative relative cost. Thus, to start a new iteration, the master problem is solved considering the new columns along with the columns from the previous iteration.

When, after solving all subproblems, no generated cutting pattern obtains a negative reduced cost, it means that the optimal solution of the linear relaxation of the master problem was reached. Then a procedure is applied to determine an integer solution. In this procedure, the homogeneous columns initially generated, and the columns added during the iterations are used in the master problem, which is solved considering the integrality constraints. Finally, the gap is calculated, which is the difference in percentage between the integer solution and the optimal solution of the linear relaxation.

It is possible to observe, in Figure 18, that no processing time limit was defined in the column generation, but in the solution of the master problem with integrality constraints, the time limit was set at 20 minutes for the random data instances and 1 hour for the real instance.

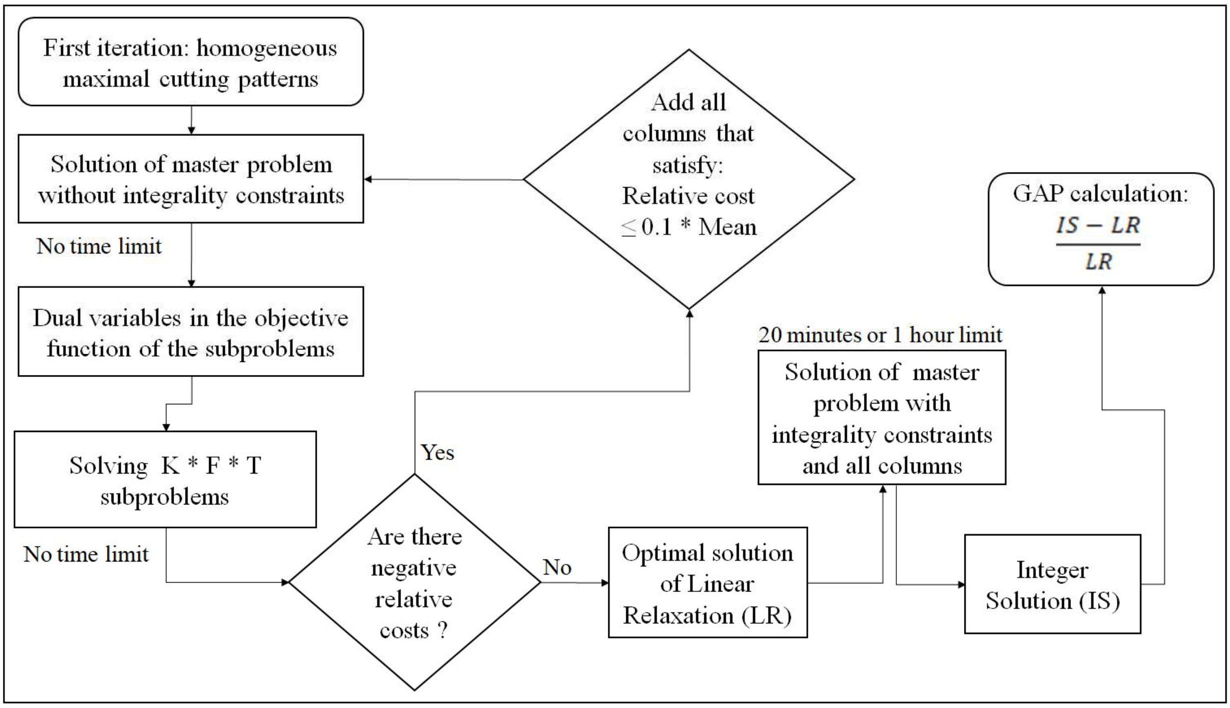

Figure 18 – Flowchart of the solution method. Source: Author.

3.4 Computational Results and Discussion

In this section, the results of applying the proposed model (1)-(7) are presented. The results include real data from the industry, in Section 3.4.1, and randomly generated data, in Section 3.4.2. The detailed data are available online on the GitHub platform, through the following link.

https://github.com/prochavetz/AMMOD-D-20-01547

Optimization Programming Language (OPL) modeling language was used to build the mathematical model, and the CPLEX solver (version 12.10) was used to solve it. Visual Basic for Applications was used to handle the company data, generate the file to be processed by the OPL, as well as to analyze the results. The computer used to solve the instances has an Intel Core i7, 64 bit processor with 16GB RAM.

3.4.1 Real Data

The instance solved considers the weekly production of the company (five periods), for a given specific week, with 53 types of objects used for the production of 176 types of items, divided into 45 subgroups (each subgroup contains compatible items and objects). In this week, 7 types of final products were produced. As explained, the company has three cutting machines (two manual and one automatic) and their production capacity, limitations in terms of item thickness and length, and the limited types of items per cutting pattern were taken into consideration.

Although in practice, the daily inventory costs of most of the items or final products are not relevant, they were considered in the model to balance inventory levels, preventing them from rising excessively. According to company specifics, inventory costs are quite low for almost all items. Items with high demands had inventory costs per period defined as 0.5% of their lengths. For items with low demand, the established costs were 1% of their lengths. In addition, for items with very low demand, the inventory cost is set to 50% of their lengths. Inventory costs for final products were defined as the sum of inventory costs for all items that comprise it. These definitions were made in consultation with the production manager. Thus, the percentage values established for each item, as well as the calculation of inventory costs for final products were considered quite reasonable by the company.

The total demand for the week studied is 13,305 items and 221 final products, which need 1,779 items to be produced. The total production capacity is 20,880 items. The average number of item types per subgroup is 3.9, and the largest subgroup has 53 item types. There is only one item type in the smallest subgroup. The average number of object types per subgroup is 1.2, and the number of object types per subgroup ranges from 1 to 3. Considering stocks of all types of items, the average difference between the maximum stock and the minimum stock is 164 items, with the smallest difference being 31 items and the largest, 3,030 items. Table 2 presents the relevant data in this instance with the minimum, maximum and mean values, as well as standard deviations.

Table 2 – Spring factory data.

Parameter	Minimum	Maximum	Mean	Standard Deviation
Object Length (mm)	1,200	7,000	6,052	875.3
Item Length (mm)	300	2,220	1,178	475.6
Item Demand (un)	4	1,200	76.0	148.6
Final Product Demand (un)	6	82	31.6	23.4
Item by Subgroup (un)	1	53	3.9	7.8
Object by Subgroup (un)	1	3	1.2	0.48
Item Stock Range (un)	31	3,030	164.0	427.9
Final Product Stock Range (un)	30	52	38.0	6.5
Item Stock Cost (% Item Length)	0.5% ; 1% ; 50%		16.9%	23.2%
Item Production Capacity (% need for items)	138% of the need for items			

Table 3 shows the quality of the solution obtained using the proposed approach. The value of the OF and its percentage difference in relation to the optimal solution of the linear relaxation (Gap) are presented. This table also presents the number of objects used, the total length of objects cut, the total length of the losses (both in millimeters) and the percentage of losses. Finally, the inventory costs (Stock Cost), the inventory usage index (Usage %) and the inventory cost index (Cost %) are shown for both items and bundles.

To find, for example, the value of the inventory usage index of items for each instance, the inventory range is calculated ($range_i = rmax_i$ - $rmin_i$) for all item types. The average stock occupation of each item type is also calculated ($rocp_i$ = $rmean_i$ - $rmin_i$), where ($rmean_i = \frac{\sum_{t=1}^{T} r_{it}}{T}$), for all item types. So, the inventory usage index is equal to ($\frac{\sum_{i=1}^{I} rocp_i}{\sum_{i=1}^{I} range_i}$. 100). The calculation is the same for the final products. The same criterion is used to calculate the inventory cost index, but referring not to the number of units in stock, but the cost of those units in stock.

Table 3 – Results of the model using real data.

Factor		Model	Factory
Solution	OF (mm)	1,574,639	-
	Gap	9.83%	-
Loss	Objects Used	3,109	3.308
	Total Cut (mm)	19,767,110	-
	Loss (mm)	470,216	-
	Loss %	2.38%	4.75%
Item Stock	Usage %	10.86%	11.5%
	Stock Cost (mm)	602,049	-
	Cost %	3.18%	51.2%
Bundles Stock	Usage %	13.16%	15.2%
	Stock Cost (mm)	502,374	-
	Cost %	1.68%	16.8%

Taking into account the size of this instance and its characteristics, the gap in this solution can be consider a good result. In section 3.4.2.2, it is clear that the biggest gaps occur in larger instances with small items, exactly the characteristics of this real instance.

At the spring company, the loss from the cutting process is measured in kilograms of steel lost, which means that the loss in objects with larger width and thickness, is more significant than the loss from smaller objects. During production in the week being studied, the loss in practice at the company was 4.75%, losing 6,657 kg of steel after cutting 3,308 objects whose total weight was 140,207 kg. The proposed mathematical model use loss calculated in millimeters, but to allow the comparison, in this case, the solution was analyzed in terms of weight: 3,960 kg of steel was lost after cutting 3,109 objects weighing a total of 165,908 kg, which represents a loss of 2.39%.

Note that, besides cutting 25,701 kg more, the model lost 2,697 kg less. The proposed solution also used 199 fewer objects and, as can be seen below, produced a total of 674 more items. To compare the solution, the percentage of losses generated by the model was compared with actual company data. In view of this, it can be concluded that the loss was reduced by 49.7%, from 4.75% to 2.39%. In the actual company solution, this reduction would represent a weekly saving about 3,307 kg of steel. When analyzing the loss of the proposed solution in terms of length, as will be done with all other instances of this chapter, 470,216 mm of steel were lost after cutting of objects measuring 19,767,110 mm in total, which represents a 2.38% loss.

Regarding the inventory usage index, the solution reaches aceptable values both for the items (10.86%) and final products (13.16%). The values of the inventory cost index are much lower, 3.18% for items and 1.68% for final products. This large difference is made possible by the large difference in inventory costs established in the model, with inventory costs of 0.5% for some units and 50% for others. This difference between the inventory usage index and the inventory cost index also means that the model was capable of avoiding stocks of high cost, accumulating inventories mostly from low-cost units, in this case, both for items and final products.

In practice, the inventory usage index of the company was slightly higher than the proposed solution, 11.5% for the items, and 15.2% for the final products. Even if the company does not measure storage costs, and therefore does not make decisions based on them, for comparison purposes only, the actual company solution was evaluated with the same criteria, obtaining an inventory cost index of 51.2% for items and 16.8% for final products.

The solution of the proposed model uses 83.71% of the total capacity, producing 17,478 items in the five periods, meeting the demand for items and final products. By comparison, the company solution in practice used 80.48% of the capacity to produce 16,804 items. As the total need for items was 15.084 units, both solutions increase inventories. In the solution of the model, production close to the demand period allowed that, even with an increased production, the inventory cost indexes remained low. Additionally, the increase in production occurs mainly in items with low stock costs that allow good cutting patterns, consequently reducing losses. Achieving this result was facilitated by considering low inventory costs. In addition, it is important to note that even though inventories have been accumulated, there is a limitation for this increase, since all types of items and final products remain, in all periods, below the maximum stock established by the company.

Table 4 shows the computational performance of the model applied to the spring factory data. The first lines show the number of subgroups of items and objects and the number of iterations required for the solution. The number of columns, considering initial columns from homogeneous patterns (Homogeneous), and the columns generated during iterations (Generated) are given next. The total time and the time per iteration (both in seconds) spent solving the relaxed master problem and subproblems (By Iteration and Total) follow. Finally, the time needed to solve the master problem with integer constraints at the end of the run and the total computer time for all stages are shown.

Table 4 – Computational performance of the model with real data.

Factor		Value
Number of Subgroups		45
Number of Iterations		16
Columns	Homogeneous	487
	Added	762
Relaxed Master Time (s)	By Iteration	189
	Total	3,031
Subproblem Time (s)	By Iteration	40
	Total	637
Integer Master Time (s)		3,874
Total Time (s)		7,543

As can be seen in Table 4, most of the 2 hours and 6 minutes total computational time was used in solving the integer master problem. The processing time of the integer master problem was limited to 1 hour, and the excess time (274 seconds) was consumed in loading the data. In the column generation, most of the processing time was spent solving the relaxed master problem. To compare the time taken to find the solution, the spring company estimates that an employee takes 42,000 seconds (11.7 hours) per week or 8,400 seconds (2.3 hours) per day to do this manually. A practical gain lies in the fact that the solution is automatically found, requiring only a few minutes for an employee to enter the data and export the solution.

3.4.2 Random Data

In order to evaluate the performance of the model (1)-(7), random instances with different characteristics generated as described in Section 3.4.2.1 were used. These instances also allowed additional managerial insights to be obtained. The results are presented in Section 3.4.2.2.

3.4.2.1 Generation of the Random Data Instances

These instances were divided into 18 groups, which differ in terms of the number of items types and object types, inventory costs, and length of the items. The characteristics of each group are shown in Table 5:

Table 5 – Groups of instances with random data.

Random Instances		Size of the Instances					
		35 Item Types; 15 Object Types; 5 Final Product Types			70 Item Types; 30 Object Types; 10 Final Product Types		
		Item Inventory Costs					
		0-5%	10-15%	20-25%	0-5%	10-15%	20-25%
Length of Items (mm)	500-1,000	G 1	G 4	G 7	G 10	G 13	G 16
	1,000-1,500	G 2	G 5	G 8	G 11	G 14	G 17
	1,500-2,000	G 3	G 6	G 9	G 12	G 15	G 18

For each of the groups, 10 instances were generated with random data, within the levels fixed above, totaling 180 instances. Each instance has, besides the characteristics of each group, four periods and three machines having the same characteristics as those used by the spring company. The other necessary information was randomly defined, based on the mean and standard deviation of the real data. The parameters considered and their ranges are shown in Table 6.

Table 6 – Range of variation of the parameters of the random instances.

Parameter	Minimum	Maximum
Item Demand	10	300
Final Product Demand	5	40
Object Length	5,200	6,800
Item Stock Range	0 ; 20 ; 45 ; 80	30 ; 55 ; 90 ; 140
Final Product Stock Range	0 – 10	15 – 20
Item by Subgroup	1	7
Item by Final Product	4	10
Object by Subgroup	50% current subgroup, 50% next subgroup	
Production Capacity	115% of Items Demand	

The demand for each item and final product occurs in only one of the four periods, which is also defined at random. The definition of minimum and maximum inventories for each item is based on their demand, so that limits are higher for stock items with high demand. The initial stock for items (r_{i0}) and final products (u_{p0}) is equal to the minimum stock $rmin_i$ and $umin_p$.

To bring the instances closer to the company's practice, the inventory levels of final products are low. The inventory cost of final products, in the same way as the real data instance, is the sum of inventory costs for all items that comprise it. Finally, it is important to state that after defining (between 4 to 10) the items that make up a final product, the quantity is also randomly defined, between 1 to 3 units of each item.

The total capacity of the machines was considered as 115% of the need for the production of items in each instance to avoid infeasibilities. The capacity is equally divided by machine and period. The criterion to define the number of objects in each subgroup is that the first object makes up the first subgroup, and then each object has 50% chance of being in the same subgroup, as the previous object, and 50% chance of forming a new subgroup.

3.4.2.2 Results of Random Instances

Table 7 below, illustrates the average gap for the 10 random instances in each of the 18 groups. It is clear that instances with the smallest items generate large gaps. This occurs because small items generate more diversities of cutting patterns. As only a part of these cutting patterns (columns) are generated, it becomes more difficult to find solutions very close to the optimal solution, compared to instances with fewer possible cutting patterns. In addition, lower gaps are achieved by smaller instances. Therefore, in a practical application with a high concentration of small items, the results from the proposed approach must be analyzed and possibly improved.

Table 7 – Percentage gap for the model with random instances.

Gap		Size of the Instances								Mean
		35 Item Types ; 15 Object Types ; 5 Final Products Types				70 Item Types ; 30 Object Types ; 10 Final Products Types				
		Item Inventory Costs								
		0-5%	10-15%	20-25%	**Mean**	0-5%	10-15%	20-25%	**Mean**	
Length of Items (mm)	500-1,000	1.03%	1.26%	1.50%	**1.26%**	2.63%	2.36%	2.05%	**2.47%**	**1.81%**
	1,000-1,500	0.50%	0.66%	0.69%	**0.62%**	0.62%	0.68%	0.83%	**0.71%**	**0.66%**
	1,500-2,000	0.45%	0.56%	0.58%	**0.53%**	0.49%	0.58%	0.67%	**0.58%**	**0.55%**
Mean		**0.66%**	**0.83%**	**0.92%**	**0.80%**	**1.25%**	**1.21%**	**1.19%**	**1.21%**	**1.01%**

Table 8 shows the mean percentage loss for each group of instances. Since smaller items allow better combinations in the cutting patterns, instances with the smallest items generate smaller losses. Additionally, for low inventory costs, lower are the losses. With low inventory costs, stock levels may be higher, which allows better matching of items, reducing losses. Finally, note that large instances tend to have slightly large losses. Therefore, in a practical application, if possible, it is interesting to mix items of different sizes allowing better combinations and reduced losses. This is a relevant managerial insight that can be used by the production planning sector.

Table 8 – Percentage losses of the model with random instances.

Loss		Size of the Instances								Mean
		35 Item Types ; 15 Object Types ; 5 Final Products Types				70 Item Types ; 30 Object Types ; 10 Final Products Types				
		Item Inventory Costs								
		0-5%	10-15%	20-25%	**Mean**	0-5%	10-15%	20-25%	**Mean**	
Length of Items (mm)	500-1,000	0.76%	1.17%	1.52%	**1.15%**	1.00%	1.31%	1.56%	**1.29%**	**1.22%**
	1,000-1,500	2.55%	4.57%	4.10%	**3.74%**	3.08%	3.86%	4.90%	**3.95%**	**3.84%**
	1,500-2,000	6.76%	7.49%	7.31%	**7.18%**	6.85%	8.02%	9.12%	**8.00%**	**7.59%**
Mean		**3.36%**	**4.41%**	**4.31%**	**4.02%**	**3.64%**	**4.40%**	**5.19%**	**4.41%**	**4.22%**

Regarding the item stock usage index, shown in Table 9, as expected, instances with low inventory cost generated an increase in stock use. The size of instances and items does not show a clear trend in inventory levels. The values of the item stock cost index are, in general, slightly lower, with the total mean being 7.9%.

The same trend presented in Table 9 occur for the final products, both for stock usage index and stock cost index, for which the average values are 2.3% and 2.1%, respectively. For both items and final products, the cost index is lower than the usage index. This shows that the model was able, for random instances in general, to prioritize the formation of stocks of low-cost units.

Table 9 – Item Stock Usage index of the model with random instances.

Item Stock Usage		Size of the Instances								**Mean**
		35 Item Types ; 15 Object Types ; 5 Final Products Types				70 Item Types ; 30 Object Types ; 10 Final Products Types				
		Item Inventory Costs								
		0-5%	10-15%	20-25%	**Mean**	0-5%	10-15%	20-25%	**Mean**	
Length of Items (mm)	500-1,000	14.6%	4.7%	4.4%	**7.9%**	14.5%	5.5%	4.0%	**8.0%**	**8.0%**
	1,000-1,500	19.5%	8.3%	5.5%	**11.1%**	20.1%	8.4%	5.3%	**11.2%**	**11.2%**
	1,500-2,000	11.8%	7.2%	8.4%	**9.1%**	15.9%	5.9%	6.1%	**9.3%**	**9.2%**
Mean		**15.3%**	**6.7%**	**6.1%**	**9.4%**	**16.8%**	**6.6%**	**5.1%**	**9.5%**	**9.4%**

The average use of the production capacity was 76.9%. In the first period, more capacity is used (84.9%), and the lowest use of capacity occurs in the last period (70.2%). In general, the use of the production capacity varies little with the variation of the parameters, therefore, only the total mean is presented, without the details for each group of instances. A slight trend that can be observed is that the higher the inventory costs, the less capacity was used. It is a natural result since, if inventory costs are high, inventory build-up is avoided, so fewer items are produced, and less production capacity is used. It is also observed that large instances tend to use a little more production capacity.

Table 10 shows the total computational time required, on average, for each instance group. The limit for processing the integer master problem in this case is 1,200 seconds. As expected, this is directly related to the size of the instance. Instances with smaller items generate greater possibilities for cutting patterns, so these instances require more iterations to optimize and, thus, require more computing time.

Table 10 – Processing time (in seconds) of the model with random instances.

Computational Time (s)		Size of the Instances								Mean
		35 Item Types ; 15 Object Types ; 5 Final Products Types				70 Item Types ; 30 Object Types ; 10 Final Products Types				
		Item Inventory Costs								
		0-5%	10-15%	20-25%	**Mean**	0-5%	10-15%	20-25%	**Mean**	
Length of Items (mm)	500-1,000	1345	1334	1334	**1338**	1732	1762	1780	**1758**	**1548**
	1,000-1,500	1296	932	960	**1063**	1505	1411	1453	**1456**	**1259**
	1,500-2,000	317	323	484	**375**	713	363	622	**566**	**470**
Mean		**986**	**863**	**926**	**925**	**1317**	**1179**	**1285**	**1260**	**1093**

On average, 8.3 iterations were performed, and 335 columns were added. Of course, the groups of instances that require more iterations are those that generate the largest number of columns and take the longest computational times. All 60 instances with small items have reached the 20 minutes limit to solve the integer master problem, on the other hand, for the 60 instances with large items this has occurred only a 15 times. Instances with medium sized items reached the limit in 53 out of 60 instances.

For these random instances, most of the processing time of the column generation was required to solve the subproblems (average time of 114.4 seconds). The average time needed to solve the relaxed master problem is 95.6 seconds. This difference is much greater analyzing only small instances. Therefore, regarding the column generation, it is possible to note that increasing the size of the instance has a greater impact on the processing time of the relaxed master problem. Accordingly, in Section 3.4.1, for the real data instance, 82.6% of the column generation time was demanded by the relaxed master problem.

According to the results, in general, the best results occur for smaller instances, and/or with large items. Instances with small items become more complex to be solved, so they generate worse results, except for the loss. Among the measures analyzed, the loss was the one that best showed the influence of changing each parameter. The utilization of production capacity, however, varies very little among different groups of instances.

3.5 Conclusions

In this chapter, an automotive spring factory was studied in order to propose a mathematical model and use a solution method to obtain a solution that reduces inventory costs and losses in the steel bar cutting process. A mathematical model that captured practical issues of the company was developed to represent this problem. To analyze the performance of the model, tests with both real and random data were made. Results show that the proposed model worked well since, applied to real instances, it found significantly superior solutions to those achieved in practice. Also, the model was successfully validated by solving 180 instances with random data.

For its practical application, solutions were achieved in acceptable computational times, saving time and money for the company, while respecting all the specific operational restrictions and guaranteeing the applicability of the solution. In addition, losses were reduced in 49.7% using the proposed solution, generating great savings in raw materials, about 3.3 ton of steel per week.

Solving random instances was important to better understand specific aspects of the problem, enabling managers to make better decisions. It also demonstrates the robustness of the model by solving 18 different types of instances, explaining, for several factors, the influence of parameter changes. The loss was the measure that best showed the influence of the variation of each parameter. In general, in terms of solution quality (gap), better results were found with smaller instances and/or with large items. It gives a managerial insight in the sense that, in practical cases, if one considers a subset of relatively larger items, the approach proposed in this chapter will reach solutions of improved quality.

Analyzing the limitations of the approach proposed in this chapter, it can be stated that the non-consideration of setups makes it difficult to apply the model in companies that use cutting machines with relevant setup costs, such as milling cutter, punching machines and other machines that require large tool changes from one object to another. In the same way, companies with high inventory costs would require adjustments for a satisfactory operation of this model. This may be the case for companies that have expensive processes downstream of the cutting process, or that produce items with high added value. In this case, the cost of loss of raw material is not as significant as the cost of a high stock of final products. Finally, modifications would be required to implement this model in environments where the cost of using productive capacity is more relevant, such as when using or providing outsourced services. In this case, the cost of material waste competes with the cost of using capacity.

4. A 3-LEVEL INTEGRATED LOT SIZING AND CUTTING STOCK PROBLEM APPLIED TO A TRUCK SUSPENSION FACTORY

This chapter addresses a 3-level Integrated Lot Sizing and Cutting Stock Problem (3-ILSCSP) applied to the production of truck springs of a truck suspension factory. The production process requires cutting bars (objects) into springs (items) which are assembled into spring bundles (final product), and it is imperative to decrease losses of bars.

Figure 19 shows the relations between the three levels of production and the time periods (Melega *et al.* 2018) applied to the studied case. The three boxes represent the three production levels (bars, springs, and spring bundles). The curved arrows that arrive in each box represent the stock that can be kept from the period *t-1*, and the curved arrows that come out of each box represent the stock that can be kept for the period *t+1*. Vertical arrows represent demand at each production level, and horizontal arrows show the precedence relation in each level's production/purchase.

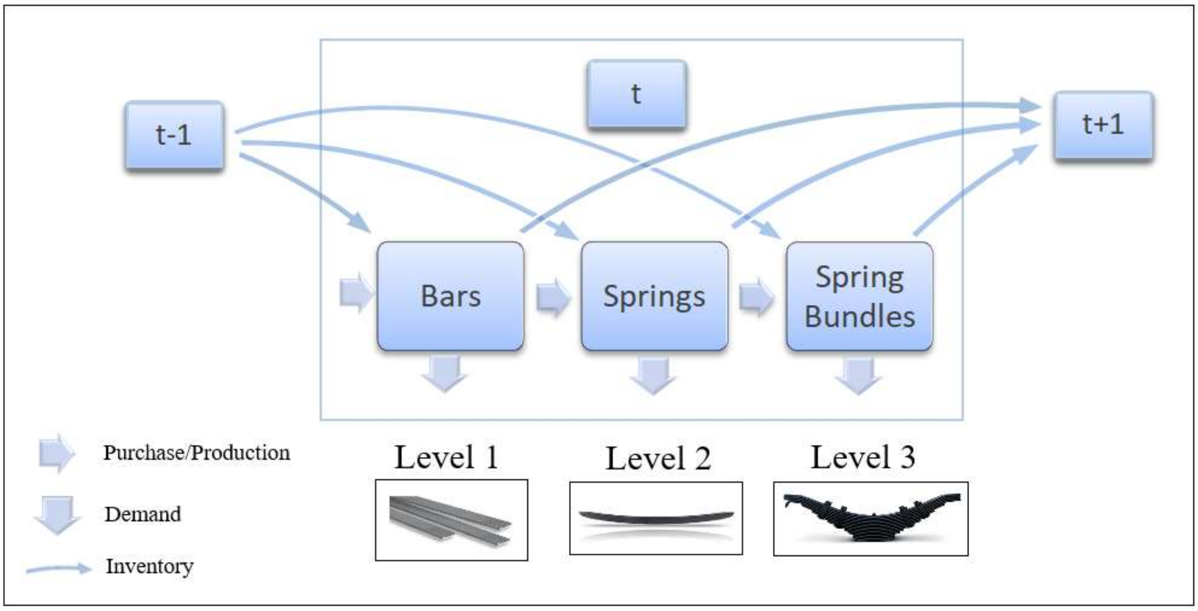

Figure 19 – Relations between three production levels over multiple time periods, applied to the spring company. Source: Author.

The contribution of this chapter to the literature is the proposal of a mixed integer mathematical model depicting a practical problem through an integrated approach. Following the future research agenda proposed in Andrade *et al.* (2021), the model is built focused on the company's medium-term issues (total period of eight months), taken into account multiple periods of time, machine capacities, the demand for bars, springs, and spring bundles, as well as costs, and inventory limits. The acquisition of all kinds of bars is a decision variable in the model. The assembly of spring bundles is analyzed along with the process of cutting springs. According to literature reviews to be analyzed in Section 4.1, this approach is very unusual. Moreover, considering as many elements as this study, no article was found by the authors.

The linear relaxation of the proposed model was solved using the Simplex Method with column generation (Gilmore and Gomory, 1961). Then, a computational package is used to obtain an integer solution. This solution method is straightforward, justified because this chapter's focus is its application rather than methodology.

Apply a scientific approach to a company belonging to one of Brazil's most relevant industries, with extensive computational testing, strengthens available literature, and current practices. The validation of this approach occurs through the solution of instances with both real and random data. The spring manufacturer's case shows the model's quality and the study's importance once it allowed for a relevant decrease in losses, which added up to 30%. By solving random instances, one can evaluate how different parameters influence results, attaining a deeper comprehension of the problem, and managerial insights that may further help with practical outcomes.

This chapter is organized as follows. In Section 4.1, a literature review of related papers is presented. Section 4.2 contains the proposed mathematical model, and Section 4.3, the solution method. In Section 4.4, the computational results are presented, and finally, Section 4.5 contains the conclusion.

4.1 Literature Review

According to the literature review carried out by Melega *et al*. (2018), of the 34 papers approaching the ILSCSP, conducted along 32 years (1985 to 2017), only three are categorized as (L1/L2/L3/M), making the complete 3-level approach in the same way as this chapter (Arbib and Marinelli, 2005; Ouhimmou *et al*. 2008; Melega *et al*. 2016). The authors have found no papers published in the following years (2018 to 2020) applying this particular approach.

In Arbib and Marinelli (2005), a gear belts company is studied to propose an integrated approach between the purchase planning of objects and the production of items and final products. The purchase of objects is modeled as a LSP considering costs of transport and storage objects, as well as production costs. In the production of gear belts, there is a one-dimensional multiperiod CSP. Since there is no demand for items, all items are kept in intermediate stock after the cutting process until the sewing process, which turns them into final products. Computational tests are performed on instances based on real data. Compared to company practice, the integrated approach reduces, on average, 43% of total costs.

The wood supply chain in a Canadian furniture company is addressed broadly in Ouhimmou *et al*. (2008). The objective is to define the procurement, inventory, transport, production, and outsourcing policy at the minimum total cost. The cutting process takes place at sawmills, and a drying process transforms items into final products. Setups and production capacities are considered in both cases. The demand occurs only for final products and is variable every period. If there is no production capacity to meet demand, final products can be purchased on the market. In addition to solving the mathematical model using CPLEX, a heuristic was developed. Computational tests are performed in fictitious instances to compare the performance of the methods based on solution quality and computational time. For the solution of the real instance, the processing time of the CPLEX became impracticable, so only the heuristic was applied.

In Melega *et al.* (2016), a theoretical study aims to propose mathematical models that deal with the ILSCSP and compare them with models in the literature. The authors also analyzed two solution methods and used different data sets to compare the results for each model. In the model that considers multiperiod and the three levels of production, the production of the various types of objects is a parameter, so the freedom of the model over the level of the objects in stock occurs through the production plan of items and final products. Production capacities and setups are considered in the production of final products. It is important to highlight that, as the model does not consider stock or demand for items, after being cut, every item must, in the same period, be processed to become a final product.

This complete 3-level approach is less performed, and studies often present greater simplifications (Melega *et al.* 2018). Accordingly, it is noteworthy that none of these 3 articles addressed, in the same way as the present study, a real case simultaneously considering demands and stock for objects, items, and final products. So, in this literature review, it is argued that although other works with similar approaches are found, this study has novelties in terms of mathematical modeling, mainly because it considers object purchases as one of the decision variables.

Analyzing the differences between these three papers and the present study in more detail, it is noted that in Arbib and Marinelli (2005), only one type of object is considered, and there is no limit on the purchase of the objects. Also, neither demand for items nor inventories of final products are considered. In Melega *et al*. (2016), the object production is not a decision variable, production capacity or inventory for items are not considered, and no real case is addressed. A complete approach is found in Ouhimmou *et al*. (2008), once a real case is treated, and there are setups and capacity limits at all levels of production. However, there is a demand only for final products. Objects and items are necessarily processed (cut or drying) until they become final products and then sold. In addition, unlike the present study, the authors could not solve the real instance using a computational optimization package.

4.2 Mathematical Model

The mathematical model presented considers the acquisition of steel bars (objects), which are cut into springs (items) and ultimately assembled into spring bundles (final products). All three productions levels admit different demands, inventory costs, and limits. For each period, it has been set a purchase limit for the bars and a production limit for the springs. Given the company's reality, no limit for spring bundles production was set. Each term used in the mathematical model is defined as follows:

Sets:

I: set of spring types $\{i = 1, \dots, |I|\}$ (index *i*);

K: set of bar types $\{k = 1, \dots, |K|\}$ (index *k*);

J: set of cutting patterns $\{j = 1, \dots, |N_K|\}$ (index *j*). Where N_k is the number of cutting patterns of bar type *k*;

P: set of spring bundle types $\{p = 1, \dots, |P|\}$ (index *p*);

T: set of periods $\{t = 1, \dots, |T|\}$ (index *t*).

Parameters:

dr_{it}: demand of spring type i in period *t*;

ds_{kt}: demand of bar type *k* in period *t*;

dp_{pt}: demand of spring bundle type p in period *t*;

α_{ijk}: quantity of spring type i produced by cutting pattern j from bar type *k;*

z_{ip}: quantity of spring type i in one unit of spring bundle type p;

$emax_t$: purchase limit of bars in period *t*;

cap_t: production capacity (in number of springs) of the cutting machines in period *t*;

c_{jkt}: cost of cutting (in mm of bar) a bar type *k* according to the cutting pattern j in period *t;*

cr_{it}: storage cost (in mm of bar) of spring type *i* in stock at the end of period *t*;

cs_{kt}: storage cost (in mm of bar) of bar type *k* in stock at the end of period *t*;

cp_{pt}: storage cost (in mm of bar) of spring bundle type *p* in stock at the end of period *t*;

$rmin_i$: minimum stock of spring type *i*;

$rmax_i$: maximum stock of spring type *i*;

$smin_k$: minimum stock of bar type *k*;

$smax_k$: maximum stock of bar type *k*;

$umin_p$: minimum stock of spring bundle type *p*;

$umax_p$: maximum stock of spring bundle type *p*;

L_k: length of bar type *k*;

l_i: length of spring type *i*.

Decision Variables:

x_{jkt}: number of bars type *k* cut according to cutting pattern *j* in period *t*;

y_{pt}: number of spring bundles type *p* produced in period *t*;

r_{it}: number of springs type *i* in stock at the end of period *t*;

s_{kt}: number of bars type *k* in stock at the end of period *t*;

u_{pt}: number of spring bundles type *p* in stock at the end of period *t*;

e_{kt}: quantity of bar type *k* purchased in period *t*.

The mathematical model is as follows:

$$min \quad \sum_{t=1}^{|T|}(\sum_{k=1}^{|K|} cs_{kt}s_{kt} + \sum_{k=1}^{|K|}\sum_{j=1}^{|N_k|} c_{jkt}x_{jkt} + \sum_{i=1}^{|I|} cr_{it}r_{it} + \sum_{p=1}^{|P|} cp_{pt}u_{pt}) \tag{14}$$

$$s.t. \quad s_{kt} = s_{k,t-1} - ds_{kt} + e_{kt} - \sum_{j=1}^{|N_k|} x_{jkt}\,, \qquad \forall\, k \in K, t \in T, \tag{15}$$

$$\sum_{k=1}^{|K|} e_{kt} \leq emax_t\,, \qquad \forall\, t \in T, \tag{16}$$

$$smin_k \leq s_{kt} \leq smax_k\,, \qquad \forall\, k \in K, t \in T, \tag{17}$$

$$r_{it} = r_{i,t-1} - dr_{it} + \sum_{k=1}^{|K|}\sum_{j=1}^{|N_k|} \alpha_{ijk} x_{jkt} - \sum_{p=1}^{|P|} z_{ip} y_{pt}\,, \qquad \forall\, i \in I, t \in T, \qquad (18)$$

$$\sum_{i=1}^{|I|}\sum_{k=1}^{|K|}\sum_{j=1}^{|N_k|} \alpha_{ijk}\, x_{jkt} \leq cap_t\,, \qquad \forall\, t \in T, \qquad (19)$$

$$rmin_i \leq r_{it} \leq rmax_i\,, \qquad \forall\, i \in I, t \in T, \qquad (20)$$

$$u_{pt} = u_{p,t-1} - dp_{pt} + y_{pt}\,, \qquad \forall\, p \in P, t \in T, \qquad (21)$$

$$umin_p \leq u_{pt} \leq umax_p\,, \qquad \forall\, p \in P, t \in T, \qquad (22)$$

$$x_{jkt} \in \mathbb{Z}_+, \qquad \forall\, j \in J, k \in K, t \in T,\qquad (23)$$

$$y_{pt} \in \mathbb{Z}_+,\ u_{pt} \in \mathbb{R}_+ \qquad \forall\, p \in P, t \in T, \qquad (24)$$

$$s_{kt}\,, e_{kt} \in \mathbb{R}_+ \qquad \forall\, k \in K, t \in T, \qquad (25)$$

$$r_{it} \in \mathbb{R}_+ \qquad \forall\, i \in I, t \in T, \qquad (26)$$

The parameter c_{jkt} stands for the cost of cutting a bar type *k* following the cutting pattern *j* in the period *t*, and it corresponds to how many material, in millimeters, is lost by the application of this cutting pattern, that is:

$$c_{jkt} = L_k - \sum_{i=1}^{|I|} \alpha_{ijk} l_i \qquad \forall\, k \in K, j \in J, t \in T. \qquad (27)$$

The mathematical model (14)-(26) is all organized around the production levels. Constraints (15)-(17) refer to level 1 (purchase of bars), level 2 (cut of springs) is addressed in constraints (18)-(20), and constraints (21) and (22) relate to level 3 (assembly of spring bundles). In the OF (14), the first term refers to level 1, minimizing the cost of stocking bars. The following two terms deal with level 2, minimizing material losses in the cutting process, and springs inventory costs, respectively. The last term deals with level 3, minimizing inventory costs for spring bundles. Constraints (15) ensure that bars demand is met and that inventory relations are maintained between periods. Constraints (16) guarantee that the purchase limit is respected for each period. Constraints (17) determine the minimum and maximum stock limits for bars. In (18), for all springs types in all periods, the inventory relations are established, ensuring that the demand for springs is met. The assurance that machines have their production capacities observed throughout the periods is given by the Constraints (19). Constraints (20) establish minimum and maximum stock limits for springs. In (21), the demand for all types of spring bundles must be met in all periods, and the inventory relations must be respected. In (22), the stock limits for spring bundles are defined.

The parameters s_{k0}, r_{i0} and u_{p0} refer to the initial stock and vary by instance, accepting any given value that sits within the established limits. Lastly, the domain of all decision variables is delineated in (23)-(26).

Importantly, the model decides the quantity in stock of a determined bar k in period t (see constraints (15)). This decision is based on the quantity of each bar to be purchased in each period, represented in the model by the variable e_{kt}. The model has more flexibility to accumulate or use bar inventory between periods and, thereby, minimize loss of material together with the storage costs. Considering e_{kt} as a variable, not as a parameter, considerably improves the results in relation to the minimization of losses (Poldi and De Araujo, 2016).

4.3 Solution Method

In line with Gilmore and Gomory (1961), Simplex method with column generation was the chosen method applied in solving the linear relaxation of the model in Section 4.2. Following, integer solutions were obtained using a computational package. In Vanzela *et al.* (2017), Melega *et al.* (2020), and others, a similar approach was adopted. The column generation procedure has been already defined in Chapter 3, so that in this chapter only specific information is presented.

The amount of subproblems solved at each iteration equals the product of the number of bar types (K) and periods (T). Also, a distinct cutting pattern (j) results from each subproblem, therefore, indexes k, j and t are fixed in each subproblem. The terms L_k, π_t and π_{kt} presented below are consequently regarded as constants. For the variable α_{ijk}, index i varies while j and k are fixed indexes. Added to the ones shown before, the terms applied to the mathematical model of the subproblems are as follows:

Sets:

G_k: Set of springs that can be produced from bar type k.

Parameters:

π_{kt}: the dual variable of Constraint (15), referring to bar type k in period t;

π_{it}: the dual variable of Constraint (18), referring to spring type i in period t;

π_t: the dual variable of Constraint (19), referring to period t.

Decision Variables:

α_{ijk}: quantity of spring type i produced by cutting pattern j from bar type k.

Below is the mathematical model of the subproblems (for each $k \in K$, and $t \in T$):

$$\min \quad L_k - \sum_{i \in G_k} \alpha_{ijk}(l_i + \pi_{it} + \pi_t) - \pi_{kt} \tag{28}$$

$$s.t. \quad \sum_{i \in G_k} \alpha_{ijk} l_i \leq L_k, \tag{29}$$

$$\alpha_{ijk} \in Z_+, \qquad \forall\, i \in G_k \tag{30}$$

In (28)-(30), the OF (28) minimizes the relative cost of the generated columns. Also, it is assured by Constraints (29) that the length of the bar type k shall not be exceeded by the sum of the lengths of the springs in the cutting pattern. In (30), the domain of variables is defined.

Similar to Chapter 3, a strategy was developed aiming at decreasing computational time for the master problem whilst maintaining the quality of the final solution. Only the columns with the lowest relative costs are included in the master problem at each iteration. Columns with relative costs less than or equal to the average value (among negative relative costs) multiplied by 0.1 are chosen. The number 0.1 was reached through early testing. The lower this number, the more columns are included once that action is based on negative values.

In short, the method goes as follows: the relaxed master problem is solved at each iteration. The OF of the subproblems is solved with the duals obtained. Columns that originated are added following formerly described criteria, leading to a new solution of the relaxed master problem. In the last iteration, the linear relaxation of the master problem is optimized. Next, an integer solution is reached solving the master problem with integrality constraints and all included columns.

Although it is a straightforward procedure, good gaps could be obtained in reasonable computational time through computational experiments. Other papers using column generation methods consider this approach when the final gap is tight enough for industry applicability (Lemos *et al.* 2021). Finally, to analyze the quality of the integer solution, the gap between the value of the OF reached in the integer problem, and that of the relaxed optimal solution is calculated.

4.4 Computational Results and Discussion

This section presents the results obtained by applying the model (14)-(26). Results using real data from the spring manufacturer are shown in Section 4.4.1, while those related to randomly generated data are shown in Section 4.4.2. Detailed data may be found online in

GitHub.com, at the following link (https://github.com/prochavetz/A-3-level-ILSCSP-Applied-to-a-Factory).

The mathematical model has been built using the Optimization Programming Language (OPL) and solved using the CPLEX solver (version 12.10). The manufacturer’s data was managed in Visual Basic for Applications (VBA), which was the tool that generated the file later processed by the OPL and used to analyze results. The instances have been solved in a computer with an Intel Core i7, 64-bit processor and 16GB RAM. In the column generation, there was no processing time limit set for these solutions. For solving the integer problem, the one-hour limit was established for real data and a twenty-minute limit was established for random data.

4.4.1 Real Data

The real data instance considers an eight-month production period (four two-month periods), allowing bar-related issues to be considered. In this case, there are 95 bar types used to produce of 269 spring types. Also 17 spring bundle types were demanded, and the number of considered subgroups are 54 (each subgroup contains compatible springs and bars). It is known that a medium-term approach implies uncertainties about the feasibility of practical application of the solution. Therefore, although the company's production capacity over a two-month period is approximately 167,040 springs, in this instance, a capacity of 160,000 springs per period, i.e. 4.2% less, was used.

Of the 95 types of bars, only 29 were sold and were considered for bar demand. The total demand for springs was 349,826 and for bars 16,699. Regarding the spring bundles, 12,699 units were demanded, which needs 72,138 springs to be produced. The total production capacity over 4 periods was 640,000 springs, and the total bar purchase limit was 200,000.

Most springs have low inventory costs for the manufacturer. However, these costs avoid an excessive increase in stock levels. The inventory cost per period was set, with assistance from the production manager, at 0.5% of its length for high demand springs, 1% for low demand springs, and 50% for considerably low demand springs. For all bars, inventory costs were set at 1% of its length. Inventory costs for spring bundles were defined as the sum of inventory costs for all springs that comprise it. The company regarded the percentages set for springs and bars and the inventory costs calculation for spring bundles as adequate.

The variation between maximum and minimum stocks ranges from 28 to 1,520, with an average of 162.7 for all spring types. The average number of spring types per subgroup is 5.0, varying between 2 to 17. The amount of bar varieties in every subgroup stands between one and four, averaging at 1.8. The relevant data in this instance is shown in Table 11, containing minimum, maximum and average values in addition to standard deviations.

Table 11 – Spring factory data.

Parameter	Minimum	Maximum	Mean	Standard Deviation
Spring Length	390	2,220	1,345	342.3
Bar Length	4,560	7,275	5,908	621.6
Spring Demand	15	1,129	325.1	222.2
Bar Demand	77	277	144.0	37.0
Bundle Demand	22	395	186.8	106.3
Spring by Subgroup	2	17	5.0	2.5
Bar by Subgroup	1	4	1.8	0.66
Spring Stock Range	28	1,520	162.7	140.6
Bar Stock Range	99	1,842	875.2	295.2
Bundle Stock Range	30	52	37.6	5.9
Spring Stock Cost	0.5% ; 1% ; 50%		2.5%	9.8%
Bar Stock Cost	1%		1.0%	0.0%
Production Capacity	152% of the need for springs			
Bar Purchase Limit	47% of the need for springs			

The solution's quality is presented in Table 12, which indicates de value of the OF and the difference in percentage from the optimal solution of the linear relaxation (Gap). It also indicates the number of bars used (Bars Used), the total length (in millimeters) both of bars cut (Total Cut) and losses (Total Loss), as well as the percentage of losses (Loss %). Last, Table 12 shows the inventory usage index (Usage %), the inventory costs (Stock Cost), and inventory cost index (Cost %) for springs, bars, and spring bundles.

The following explains, as an example, how to calculate the springs inventory usage index. First, the inventory range must be calculated ($range_i = rmax_i - rmin_i$) for all types of springs. Then, it is calculated the average stock occupation for every spring type ($rocp_i = rmean_i - rmin_i$), in which ($rmean_i = \frac{\sum_{t=1}^{T} r_{it}}{T}$). Finally, the springs inventory usage index ($\frac{\sum_{i=1}^{I} rocp_i}{\sum_{i=1}^{I} range_i}$ x 100) can be calculated. The same expression is applied for bars and spring bundles. Furthermore, the same criterion is applied to calculate the inventory cost index, not referring to the number of units in stock, but to the cost of those units in stock.

Table 12 – Solution of the model with real data.

Factor		Model	Factory
Solution	OF (mm)	30,464,855	-
	Gap	0.12%	-
Loss	Bars Used	102,796	107,382
	Total Cut (mm)	603,724,400	621,470,800
	Total Loss (mm)	20,134,255	29,653,730
	Loss %	3.34%	4.77%
Springs Stock	Usage %	8.68%	25.1%
	Stock Cost (mm)	3,035,211	-
	Cost %	5.02%	34.9%
Bars Stock	Usage %	0.00%	27.9%
	Stock Cost (mm)	6,834,112	-
	Cost %	0.00%	27.9%
Bundles Stock	Usage %	1.48%	16.5%
	Stock Cost (mm)	461, 277	-
	Cost %	0.23%	21.7%

This is a result of great quality, considering the proximity of the optimal solution, represented by the low value of the gap. The gap is the percentage difference between the integer solution, obtained at the end of processing, and the optimal solution of the linear relaxation. Analyzing the influences on the gap, in section 4.4.2.2, it is noted that low inventory costs generate low gaps, as well as the presence of large springs, two characteristics of this real instance. Even so, considering the size of this instance, a considerably larger gap could be expected.

The values for the inventory usage (8.7%) and cost (5.0%) indexes of springs are all acceptable, taking into account the size of this instance. The inventory usage index (1.5%), and inventory cost index (0.2%) for the spring bundles are low, according to the company's intention. In both cases, the inventory cost index is lower than the inventory usage index, demonstrating that the model was able to accumulate stocks, preferably from low-cost units. Note that the inventory cost and usage of bars were equal to the minimum level in all periods.

The inventory usage levels practiced by the company in the analyzed periods were high, considering that this is a company policy. The percentage was 25.1% for springs, 16.5% for spring bundles, and 27.9% for bars. Regarding the inventory cost index, it is important to state that, when this solution was praticed, the company did not measure inventory costs and did not make decisions based on it. For comparative purposes only, the inventory cost index of the practiced solution was calculated. The value was 34.9% for springs, 21.7% for spring bundles, and 27.9% for bars. Considering that the stock cost of bars is fixed at 1% of the length for all varieties, it is natural that there is no variation between the two indexes related to the bars. Although they are high, inventories of spring bundles are at the lowest levels, according to the company's intention.

The studied company measures the loss in steel kg, which means that the loss of smaller width and thickness bars is less significant than the loss of larger bars. During these 8 months, 107,382 bars were cut in the practiced solution, weighing a total of 7,950,389 kg, of which 372,552 kg were lost, or 4.69%. The loss of the proposed solution, both for the real instance and for the random instances, is measured in bar millimeters. However, to allow the comparison, only in this case, it was calculated the loss in kg of steel. Thus, 102,796 bars are cut in the proposed solution, whose total weight is 9,769,582 kg, with 366,205 kg of steel being lost, or 3.75%.

Although the proposed solution presents a small reduction in the lost weight (6,347 kg), it cuts 1,819,193 kg more and uses 4,586 bars less. As can be seen below, the proposed solution also produces 28,464 springs more than the practice of the company. Analyzing the percentage of loss in each solution, a reduction of 20% is noted (from 4.69% to 3.75%), which represents, in the solution practiced by the company, a saving of more than 74,500 kg of steel in 8 months.

Analyzing the loss in millimeters, as will be done in all other instances of the chapter, the loss of the proposed solution was 3.34%, as 20,134,255 mm of bar were lost, in a total length cut of 603,724,400 mm. Even though the company does not measure the loss in this way, analyzing the solution practiced in millimeters, 621,470,800 mm of bar were cut, losing 29,653,730 mm, or 4.77%.In this case, the loss was reduced by 30%.

The computational performance of the model applied to the company's data is presented in Table 13. The first and second lines consist of the number of subgroups and iterations necessary to reach the optimal solution, respectively. Next is the number of columns, both initial columns generated from homogenous patterns (Homogenous), and columns originated from iterations (Added). Next are shown, in seconds, the total time and time per iteration that took to solve the relaxed master problem and subproblems (By Iteration and Total). The last two lines consist of the total time required to solve the master problem with integer constraints at the end of the run (Integer Master Time), and the sum of computational time for all stages of processing (Total Time).

Table 13 – Computational performance of the model with real data.

<table>
<tr><th colspan="2">Factor</th><th>Value</th></tr>
<tr><td colspan="2">Number of Subgroups</td><td>54</td></tr>
<tr><td colspan="2">Number of Iterations</td><td>20</td></tr>
<tr><td rowspan="2">Columns</td><td>Homogeneous</td><td>269</td></tr>
<tr><td>Added</td><td>409</td></tr>
<tr><td rowspan="2">Relaxed Master Time (s)</td><td>By Iteration</td><td>43.7</td></tr>
<tr><td>Total</td><td>874.5</td></tr>
<tr><td rowspan="2">Subproblem Time (s)</td><td>By Iteration</td><td>22.4</td></tr>
<tr><td>Total</td><td>448.9</td></tr>
<tr><td colspan="2">Integer Master Time (s)</td><td>3,691.8</td></tr>
<tr><td colspan="2">Total Time (s)</td><td>5,015.2</td></tr>
</table>

As shown in Table 13, the majority of the total processing time, which equals to 1 hour and 24 minutes, was consumed in solving the integer master problem. This processing time had been limited to one hour, being that the remaining 91.8 seconds were spent in data loading. The relaxed master problem took the largest amount of processing time of the column generation. The company makes the bars purchase decision and the elaboration of cutting patterns manually. The time taken for the purchasing decision is not recorded, but the estimated time to generate the cutting patterns in an 8 months period, is about 350 hours. Taking that into account, a practical gain can be seen when reaching a solution is an automatic process that demands a few minutes to enter the data and export the solution.

Regarding the use of production capacity, the proposed solution uses 67.6% of the total capacity, producing 432,821springs over the four periods. The production in the last period is the largest (70.7%). The use of the bar purchase limit was 59.7%, buying 119,495 bars. Similarly, the last period is the one with the large quantity of bars purchased (61.7%).In the solution practiced by the company, 404,357 springs were produced in the 4 periods, using 63.15% of the total production capacity. Moreover, 67.8% of the bars purchase limit was used in the purchase of 135,685 bars.

With bars purchases being a model decision, a strategic component is added to this process. Unlike the company's practice, it buys not only bars whose stock level is low but allows for larger purchases of bars that enable good combinations with high demand springs. This approach provides valuable practical gains because, in addition to reducing the stock of bars, it helps to reduce losses and the total bars purchased.

4.4.2 Random Data

Section 4.4.2.1 presents the criteria used in generating random data instances, which allow assessing the quality of the model (14)-(26) when tested under different conditions. Such tests also allow managerial insights from a better understanding of the problem and the model. Results are described in Section 4.4.2.2.

4.4.2.1 Criteria for generation of the Random Data Instances

Random data instances were split among 18 groups varying in the number of spring, bar, and spring bundle types, inventory costs, and lengths of springs. Table 14 presents each group's characteristics:

Table 14 – Groups of instances with random data.

Random Problems		Size of the Instances					
		35Spring Types | 15Bar Types |5Bundle Types			70 Spring Types | 30 Bar Types |10Bundle Types		
		Spring Inventory Costs					
		0-5%	10-15%	20-25%	0-5%	10-15%	20-25%
Length of Springs (mm)	500-1,000	G 1	G 4	G 7	G 10	G 13	G 16
	1,000-1,500	G 2	G 5	G 8	G 11	G 14	G 17
	1,500-2,000	G 3	G 6	G 9	G 12	G 15	G 18

Each one of these groups had 10 instances generated with random data within the limits described in Table 14, there being a total of 180 instances. For each instance, four production periods were considered. Based on the real data's mean and standard deviation, the remaining information needed was randomly determined. Table 15 shows the parameters considered and their ranges.

The demand for springs varies randomly between 50 and 400 units per period. Lower and upper limits for inventories of each spring were determined based on the demand for them; therefore, high demand springs have a higher stock limit. The initial stock for springs, r_{i0}, equals the minimum stock $rmin_i$, and the same applies for spring bundles with initial stocks u_{p0} equivalent to $umin_p$. The initial stock of a bar (s_{k0}), in all instances, is equal to 10% of the sum of the demand for all the springs it can produce, divided equally among all the bar types in the same subgroup. The cost of stocking bars is 1% of the length.

Table 15 – Range of variation of the parameters of the random instances.

Parameter	Minimum	Maximum
Spring Demand	50	400
Bar Demand	0	50% Initial Stock
Bundle Demand	10	190
Bar Length	5,200	6,800
Bar Stock Range	Equal Initial Stock	130% Initial Stock
Spring Stock Range	0 ; 20 ; 45 ; 80	30 ; 55 ; 90 ; 140
Bundle Stock Range	0 – 10	15 – 20
Spring by Bundle	4	10
Spring by Subgroup	1	7
Bar by Subgroup	50% current subgroup, 50% next subgroup	
Production Capacity	115% of the need for springs	
Bar Purchase Limit	40% of the need for springs	
Bar Initial Stock	10% of the demand of all possible springs to produce	

Analogous to the real data instance, inventory costs of spring bundles correspond to the sum of inventory costs for each spring it is comprised of. It must be mentioned that after the definition of the four to ten types of springs that will compose a bundle, it is also randomly established how much of every spring will be used, a number varying from one to three units.

The total need for spring's production is calculated, considering the use to produce of spring bundles, and to meet the demand for springs. To avoid infactibility, the limit on the purchase of bars was defined as 40% of this value, and the total production capacity, 115%. The number of bars pertaining to each subgroup is determined by the following parameter: the first bar is assigned to the first subgroup and each bar that follows has a 50% probability of being in the same subgroup and another 50% probability of being assigned to a new subgroup.

4.4.2.2 Results of Random Instances

Table 16 illustrates the average gap for the 10 random instances in each of the 18 groups. The result, in general, is good as the total average is 0.76%. Note that instances with small springs generate larger gaps, especially when combined with high inventory costs. This is because smaller springs generate more possible cutting patterns, and hence, the lower bound obtained from the linear solution relaxing the integrality constraint can potentially be less tight compared to integer solutions generated by the solution method. Also, lower gaps are achieved by smaller instances and smaller inventory costs. For that reason, in a practical case with a large concentration of small springs, especially if inventory costs are high, results originated by this approach should be evaluated and perhaps improved.

Table 16 – Percentage gap for the model with random instances.

Gap		Size of the Instances								Total Mean
		35 Spring Types ; 15 Bar Types ; 5 Bundle Types				70 Spring Types ; 30 Bar Types ; 10 Bundle Types				
		Spring Inventory Costs								
		0-5%	10-15%	20-25%	**Mean**	0-5%	10-15%	20-25%	**Mean**	
Length of Springs (mm)	500-1,000	0.83%	1.20%	2.50%	**1.51%**	1.04%	1.76%	2.72%	**1.84%**	**1.68%**
	1,000-1,500	0.15%	0.35%	0.54%	**0.34%**	0.29%	0.55%	0.58%	**0.47%**	**0.41%**
	1,500-2,000	0.09%	0.20%	0.30%	**0.20%**	0.09%	0.23%	0.34%	**0.22%**	**0.21%**
Mean		**0.36%**	**0.59%**	**1.11%**	**0.68%**	**0.48%**	**0.85%**	**1.21%**	**0.85%**	**0.76%**

The average loss percentage for each group of instances is presented in Table 17. Smaller losses derive from the instances with the smallest springs, once using smaller springs provides better combinations for the cutting patterns. It can be seen that the lower the inventory costs, the lower the losses, since stock levels may be higher for low inventory costs, favoring a better match for springs and reducing losses. Finally, within the variation proposed in this study, the size of the instance does not seem to affect the loss significantly. Consequently, springs of different sizes should be considered whenever possible in practical applications, providing better combinations and reduced losses. The production planning sector may adequately apply this managerial insight.

Table 17 – Percentage losses of the model with random instances.

Loss		Size of the Instances								Mean
		35 Spring Types ; 15 Bar Types ; 5 Bundle Types				70 Spring Types ; 30 Bar Types ; 10 Bundle Types				
		Spring Inventory Costs								
		0-5%	10-15%	20-25%	**Mean**	0-5%	10-15%	20-25%	**Mean**	
Length of Springs (mm)	500-1,000	0.39%	0.73%	0.93%	**0.68%**	0.37%	0.54%	0.65%	**0.52%**	**0.60%**
	1,000-1,500	2.28%	2.32%	2.59%	**2.40%**	2.04%	2.50%	2.74%	**2.43%**	**2.41%**
	1,500-2,000	6.04%	6.12%	6.79%	**6.32%**	6.68%	6.84%	6.73%	**6.75%**	**6.53%**
Mean		**2.90%**	**3.05%**	**3.44%**	**3.13%**	**3.03%**	**3.29%**	**3.37%**	**3.23%**	**3.18%**

Predictably, in relation to the spring stock usage index, instances with low inventory costs provoked an increase in stock usage, as seen in Table 18. No clear trend in inventory levels can be connected to the size of instances and springs. The spring stock cost index generally has somewhat lower values, with a total average of 3.8%. This variation between the usage and the cost indexes indicates the model's capability to prioritize stocking units at a lower cost.

For the stock usage and stock cost indexes, it was possible to notice the same trend seen in Table 18 regarding spring bundles. For one as for the other, the total average amounts to 3.1%. The bars stock usage is 0% for all instances, what shows that the decision on the purchase of bars was effective in reducing the inventory of bars, also in random instances.

Table 18 – Spring Stock Usage index of the model with random instances.

Spring Stock Usage		Size of the Instances								Mean
		35 Spring Types ; 15 Bar Types ; 5 Bundles Types				70 Spring Types ; 30 Bar Types ; 10 Bundles Types				
		Spring Inventory Costs								
		0-5%	10-15%	20-25%	**Mean**	0-5%	10-15%	20-25%	**Mean**	
Length of Springs (mm)	500-1,000	5.3%	4.5%	2.1%	**4.0%**	6.2%	2.8%	2.1%	**3.7%**	**3.8%**
	1,000-1,500	10.9%	3.6%	2.0%	**5.5%**	10.2%	3.1%	2.0%	**5.1%**	**5.3%**
	1,500-2,000	7.6%	4.1%	2.1%	**4.6%**	8.8%	3.2%	3.0%	**5.0%**	**4.8%**
Mean		**7.9%**	**4.1%**	**2.0%**	**4.7%**	**8.4%**	**3.0%**	**2.4%**	**4.6%**	**4.6%**

The use of production capacity was, on average, 84.2%, the results, in this case, being quite stable. A slight increase in the use of production capacity is noticed when inventory costs are low. The total average of use of the bar purchase limit was 62.2%. Instances with larger springs require more bars, so it is natural that the purchase need is greater for these instances. In addition, the increase in the size of the instances generates a little increase in the bar purchase.

In Table 19, it is possible to observe the total computational time demanded, on average, by each instance group. As anticipated, it directly corresponds to the size of the instance. Simultaneously, the smaller the springs in an instance, the more iterations are needed for optimization and, consequently, the computational time is higher.

Table 19 – Processing time (in seconds) of the model with random instances.

Computational Time (s)		Size of the Instances								Mean
		35 Spring Types ; 15 Bar Types ; 5 Bundles Types				70 Spring Types ; 30 Bar Types ; 10 Bundles Types				
		Spring Inventory Costs								
		0-5%	10-15%	20-25%	**Mean**	0-5%	10-15%	20-25%	**Mean**	
Length of Springs (mm)	500-1,000	1260	1242	1252	**1251**	1331	1328	1308	**1322**	**1287**
	1,000-1,500	644	1227	1120	**997**	1286	1281	1284	**1284**	**1141**
	1,500-2,000	30	170	136	**112**	62	310	468	**280**	**196**
Mean		**645**	**880**	**836**	**787**	**893**	**973**	**1020**	**962**	**875**

The number of performed iterations is on average 11.8, while added columns average at 124. Undoubtedly, more computational time are required for the groups of instances that generate more columns and demand more iterations. Instances with small springs have, in their totality (60 instances), achieved the limit of 20 minutes to solve the integer master problem. In instances with larger springs, this event has taken place in only 5 of the 60 instances. Finally, the processing time limit was reached in 54 instances with medium springs.

For these random instances, solving the subproblems took an average of 52.3 seconds, or most of the processing time of the column generation, whereas solving the relaxed master problem took an average of 7.5 seconds. When only small instances are considered, this difference is substantially greater. Hence, it may be observed that, regarding column generation, increasing the instance's size imposes a higher impact on the processing time for the relaxed master problem. Thus, Section 4.4.1 shows that 66.1% of column generation time for the real data instance was used in solving the relaxed master problem.

Best results derive overall from either smaller instances and/or instances with larger springs. Instances with smaller springs have more complex resolutions and generate worse results, apart from the loss. Among the considered measures, the one that most demonstrated the influence of each parameter was the gap, as opposed to the utilization of production capacity, which varied little within different groups of instances.

4.5 Conclusions

This chapter presented the study of a truck suspension manufacturer with the objective of reducing its inventory costs and losses in the process of cutting steel bars. An integrated 3-level approach is proposed considering the company's reality in a medium-term horizon. The performance of the model was evaluated through tests conducted with real and randomly generated data. The objective of this chapter has been effectively achieved, as shown by the model's results. When applied to real data, solutions were better than those carried out in practice. Solving 180 random instances has also demonstrated the model's performance under several conditions.

The integrated and complete approach that was carried out in this study enabled an excellent quality of the global result. The main measures were significantly reduced in comparison to the solution practiced by the company: losses, stocks levels of bars, springs, and spring bundles, as well as the number of bars purchased and used. These results have managed to save the manufacturer a great amount of time and money. The required computational time is acceptable while observing all operational restrictions imposed and assuring the applicability of the solution. Furthermore, the proposed solution reduces losses (30%) and raw material waste (about 75 tons of steel in eight months). Along with economic aspects, there is also a positive ecological repercussion derived from this considerable decrease.

Applying the model to random instances enabled a better comprehension of specific elements of the problem, allowing for improved managerial decision-making. Besides, the performance of the model could be demonstrated through its resolution with 18 distinct groups of instances and the interpretation, for several factors, of how changes in parameters influence results. The measure which best indicated the effect of a change in each parameter was the gap. Overall, regarding the gap (quality of solution), the best results occurred within instances that were smaller and/or had large springs. This factor provides to the managers the understanding that when a subset of larger springs is contemplated in practical situations, the approach hereby presented should promote greater quality solutions.

The consideration of bar purchases (e_{kt}) as a variable has shown importance in the good result obtained, especially in the practical case. This allows purchases to be made not only considering the stock levels of each bar, but strategically, preferring the purchase of bars that enable better combinations, reducing losses, bars inventory, and the number of bars purchased.

5. MATHEMATICAL MODELING TO OPTIMIZE THE HARDENING PROCESS IN AN AUTOMOTIVE SPRING FACTORY

The goal of this chapter is to present the proposal of a mathematical model to optimize the hardening process in an automotive spring factory, maximizing the assignment of springs in the hardening furnace. This is an approach little considered in the literature, both for its application to a hardening furnace and for the proposed methodology.

The items (springs) that make up each type of spring bundle require hardening, among other heat treatments, for the steel to acquires the required properties. Due to the high temperature, hardening furnaces consume a lot of energy during operation. Furthermore, the decision of which items, among the various types, to insert in the furnace at each assignment is complex. This decision impacts directly on the productivity of the furnace and the time required for operation, i.e. energy expenditure. Details of this problem are explained in Section 2.3.

Aiming at saving resources, the approach proposed in this study looks to optimize the occupation of the hardening furnace at each assignment. The problem is treated as a one-dimensional CSP. To consider the specific aspects of the problem in practice, explained in Section 2.3, the problem is modeled based on an arc flow formulation. This is an innovative study because no similar approaches have been found in the literature.

The main contribution of this chapter lies in the proposed mathematical model, based on the arc flow model to solve a real-life problem. This modeling allows the location of items in the cutting pattern, which is important for the problem studied here because the items need to be supported on beams inside the furnace. The practical application of this study is unprecedented because it treats the item assignment in a hardening furnace as a CSP. The approach is validated by solving an instance with real data and 180 instances with random data. This allows the analysis of the influence of different parameters on the result. Finally, the resolution of the instance provided by the spring company produced a significant increase in the daily production of the furnace, demonstrating the quality of the model and the relevance of the work.

This chapter is divided into four sections. Section 5.1 contains the literature review, presenting studies on related topics. Then, Section 5.2, on presenting the mathematical model, shows the adaptations made to handle the assignment of items to the furnace as a one-dimensional CSP using the arc flow model. The computational results obtained with real and random data are presented in Section 5.3 and finally, Section 5.4 contains the conclusions.

5.1 Literature Review

Although CSP is a classic combinatorial optimization problem, widely used in several approaches, no CSP applications were found in the automotive industry to optimize the use of tempering furnaces.

Among studies of spring manufacture, diverse applications are found, including the comparison of the performance between springs of different materials (Al-Qureshi, 2001; Kaiser *et al.* 2011), studies to predict the generation of heat in springs (Luo *et al.* 2005), optimization of steel bar cutting for spring production (Andrade *et al.* 2021), mathematical models for fatigue prediction in steel springs (Aggarwal *et al*. 2006) and data management systems for designing springs (Peng and Trappey, 1996).

Studies aimed at optimizing furnaces, including hardening, generally deal with the adjustment of furnace parameters and the influence on product quality. Applications are found in the food industry (Banooni *et al*. 2009; Kong *et al*. 2010; Omolola *et al*. 2018; Özden and Kiliç, 2020), furnaces for the curing process (Ashrafizadeh *et al*. 2012; Glick and Shareef, 2019), hardening furnaces (Cruz *et al*. 2005; Penha *et al*. 2011; Pimenta *et al*. 2015; Pimenta *et al*. 2016), furnaces for power generation (Han *et al*. 2019) and other industrial applications (Yang *et al*. 2014; Pask *et al*. 2014; Lei *et al*. 2017).

Among these studies, some stand out. Cruz *et al.* (2005) evaluated several control variables of a hardening furnace in a steel plant, seeking to improve the quality of two types of steel. Penha *et al.* (2011) analyzed thermal residual stresses, comparing the results of oil and vacuum hardening furnaces. Lei *et al.* (2017), used an Analytic Hierarchy Process (AHP) and fuzzy logic in an integrated way to optimize and evaluate the combustion performance of industrial furnaces, through the adjustment of furnace parameters. Özden and Klic (2020) compared the performance of Artificial Bee Colony, Gravitational Search, Symbiotic Organisms Search and Neural Network techniques to adjust the parameters of an oven drying eggplants.

Pimenta *et al.* (2015) and Pimenta *et al.* (2016) are similar approaches to this study, as they deal with hardening furnaces for the manufacture of springs in the automobile industry. However, the objective of the studies diverges from the proposal of this chapter, since the authors used the simplex method and statistical methods to adjust parameters of the hardening furnace in order to improve the mechanical properties of the items (coil springs for automobiles).

Note that there are papers aimed at controlling furnaces using optimization techniques. In addition, studies set in the automobile spring industry were found, aiming at improving the hardening furnace. However, the approaches are quite different from that which is proposed in this chapter. Among the furnace studies, none addresses the assignment of the items to optimize the use of the furnace space. Furthermore, no approach found addresses a CSP or uses arc flow modeling.

5.2 Mathematical Model

In this section, the proposed mathematical model to represent the problem is presented. The problem of assignment of items to the hardening furnace is treated as a one-dimensional CSP and modeled based on an arc flow formulation.

In a CSP, a cutting pattern corresponds to a particular way of cutting an object to produce smaller items. In this study, a cutting pattern represents a particular way of placing items in the furnace. The object size is equivalent to the width of the furnace. After inserting the items in the furnace following a given cutting pattern, the beams move these items along the furnace length, freeing space for a new assignment (cutting pattern) to be started. In this way, throughout production, the entire length of the furnace is occupied by items and, as one assignment finishes processing, new items can be assigned. Wolsey (1977) and Valério de Carvalho (2002), among others, used the arc flow model to represent the one-dimensional CSP.

To describe the problem using an arc flow model, a set of identical objects is considered (each representing an assignment in the furnace that is performed between two consecutive steps) with several types of items to be inserted in each object. Each item entered provides a certain margin of gain. The goal is to maximize the gain achieved by the assignment of all items. In this way, the furnace width (object size) is approximated by a number of equidistant nodes, $b = 1, \ldots, L$, where L is the total furnace width. The nodes represent parts of the same size across the width of the furnace, the only differentiation being those nodes that represent where the beams are located. The greater the number of nodes considered in any given instance, the greater its accuracy is, therefore, the smaller the rounding errors. However, the greater the number of nodes considered, the greater the complexity and processing time required for the solution of the instance.

Arcs are used to connect a starting node *d* to an ending node *e*. Each arc represents the assignment of an item *i* in a section (*d* – *e*) of the furnace and the distance between the nodes (*e* – *d*) is equal to the length of the item (l_i). It is important to point out that, in addition to these explained arcs, which represent the assignment of a type of item in a specific section of the furnace, unitary arcs are also created, which represent an empty space in the furnace and make the model feasible.

Figure 20 illustrates the arc flow model as explained. It represents Assignment 1, illustrated in Figure 16, Section 2.3.1. Each node represented by a black color is the part of the furnace that does not have a mobile beam. The gray dots represent parts where the mobile beams are located. The unitary arcs represent the empty assignment spaces and, in this case, they are located to the left of item 1, between items 1 and 4, between items 4 and 5 and to the right of item 5. Both in the real instance as in the random instances, items with unitary length were not considered, as this makes no sense in practice.

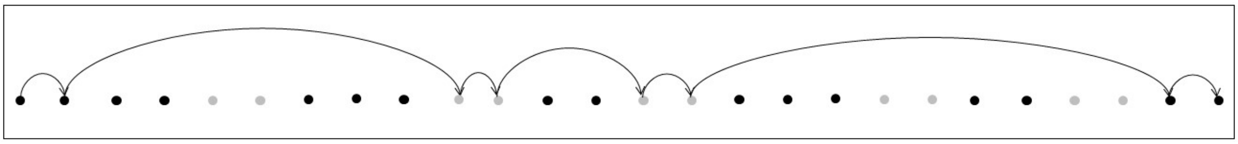

Figure 20 – Graphical representation of the arc flow formulation. Source: Author.

This modeling is considered adequate, as it is indexed by position, and as explained, the position of the item within the cutting pattern is especially important in this case, since it needs to be supported on the beams. The time horizon considered in the model is 1 period, which represents 1 day of production.

The model proposed in this study consider additional elements in relation to studies in the literature that consider arc flow models, like the setups between formulas, bender limits, time limit on production and availability of items. The following are the terms used in the mathematical model:

Sets:

I: set of item types $\{i = 1, \dots, |I|\}$ (index *i*);

K: set of furnace assignments $\{k = 1, \dots, |K|\}$ (index *k*);

T: set of formulas (temperature x speed) $\{t = 1, \dots, |T|\}$ (index *t*).

Parameters:

d_i: demand for item type i;

b_i: availability of item type i;

m_i: margin of gain of item type i;

nP: limit of parabolic bent items in an assignment;

nC: limit of conventional bent items in an assignment;

L: furnace width;

l_i: length of item type i;

pt_t: production time of an assignment in formula t;

st_t: time lost in the setup for the formula t;

TT: total time available for production in one day.

Sets and subsets of arcs:

A: viable arcs, which represent items to be assigned to nodes in the furnace. An arc (d, e) is equivalent to an item where $l_i = e - d$;
$A^t \in A$: viable arcs in formula t. Such that $\mathrm{A}^1 \cup \mathrm{A}^2 \cup \ldots \cup \mathrm{A}^\mathrm{T} = \mathrm{A}$;

$P \in A$: arcs representing parabolic bent items;

$C \in A$: arcs representing conventional bent items;

$N \in A$: arcs representing unbent (straight) items. As each item only has one type of bend: $P \cap C = \{\emptyset\},\ P \cap N = \{\emptyset\},\ N \cap C = \{\emptyset\}$;

$U \in A$: arcs of size $l = 1$, which represent an empty space in the furnace. L unit arcs are created, one for each node of the furnace.

Variables:

X_{de}^{kt}: 1 if the arc (d, e) is used in assignment k, in formula t; 0 otherwise;

Z_{kt}: 1 if assignment k is used in formula t; 0 otherwise;

W_t: 1 if formula t is used; 0 otherwise.

The mathematical model proposed to represent the problem is presented below:

$$\text{Max} \quad \sum_{i=1}^{|I|} m_i \left(\sum_{t=1}^{|T|} \sum_{k=1}^{|K|} \sum_{(d,d+l_i) \in A^t} X_{d,d+l_i}^{kt} \right) \tag{31}$$

$$\text{s.t.} \quad Z_{kt} \leq W_t \qquad t = 1, \ldots, T, k = 1, \ldots, K \tag{32}$$

$$\sum_{t=1}^{|T|} \sum_{k=1}^{|K|} \sum_{(d,d+l_i) \in A^t} X_{d,d+l_i}^{kt} \geq d_i \qquad i = 1, \ldots, I \tag{33}$$

$$\sum_{t=1}^{|T|} \sum_{k=1}^{|K|} \sum_{(d,d+l_i) \in A^t} X_{d,d+l_i}^{kt} \leq b_i \qquad i = 1, \ldots, I \tag{34}$$

$$\sum_{t=1}^{|T|}\sum_{k=1}^{|K|} pt_t Z_{kt} + \sum_{t=1}^{|T|} st_t W_t \leq TT \tag{35}$$

$$\sum_{(d,e)\in P} X_{de}^{kt} \leq nP \qquad t = 1, \dots, T, k = 1, \dots, K \tag{36}$$

$$\sum_{(d,e)\in C} X_{de}^{kt} \leq nC \qquad t = 1, \dots, T, k = 1, \dots, K \tag{37}$$

$$-\sum_{(d,e)\in A^t} X_{de}^{kt} + \sum_{(e,f)\in A^t} X_{ef}^{kt} = \begin{Bmatrix} Z_{kt} & if\ e = 0 \\ -Z_{kt} & if e = L \\ 0 & otherwise \end{Bmatrix} \qquad t = 1, \dots, T, k = 1, \dots, K \tag{38}$$

$$X_{de}^{kt} \in \{0, 1\} \qquad t = 1, \dots, T, k = 1, \dots, K, \forall (d, e) \in A \tag{39}$$

$$Z_{kt} \in \{0, 1\} \qquad t = 1, \dots, T, k = 1, \dots, K \tag{40}$$

$$W_t \in \{0, 1\} \qquad t = 1, \dots, T \tag{41}$$

In model (31) – (41), the OF (31) maximizes the sum of the margin of gain obtained from the production of all items. The constraints in (32) ensure that there is only production of a formula if a setup for that formula has been made. In constraints (33), the production of a type of item, in all assignments, in all formulas, is at least equal to its demand. (34) guarantees that the assignments of all item types do not exceed their availability (intermediate stock). The constraints in (35) limit the time used to the total time available in a day: the first term represents the time spent on production, and the second term, the time spent on setups. In this way, the processing time of the items in each assignment and their consumption of available resources are weighted. Constraints (36) and (37) ensure that the capacity limit of the benders (parabolic and conventional, respectively) is respected at each assignment. The constraints in (38) present the set of constraints related to the arc flow approach. If an assignment k is made in formula t (Z_{kt} = 1), then the sum of all arcs leaving and arriving at each node is zero for all intermediate nodes, this sum being one for the initial node (one arc leaves and no arc arrives), and "-1" for the end node (an arc arrives and no arc leaves). On the other hand, if formula t is not used in an assignment k (Z_{kt} = 0), no item is assigned, so the sum in all cases is 0. Finally, (39) - (41) define the domains of the variables.

Besides other aspects previously cited, the consideration of the variable X_{de}^{kt} as binary is also an innovation compared to other studies that address the one-dimensional CSP using arc flow models. In this application, as only one item can be assigned to a furnace location, and each assignment has it specific constraints, due to formula differences, bender limits, this variable cannot be considered as an integer. Consider X_{de}^{kt} as binary increases, above all, the complexity for solving the problem.

5.3 Computational Results and Discussion

In this section, the results of using the model (31) - (41) are presented. The instances include actual company data in Section 5.3.1 and randomly generated data in Section 5.3.2. The detailed data are available online on the GitHub platform, through the following link.

https://github.com/prochavetz/Mathematical-modeling-to-optimize-the-hardening-process-in-a-factory

The Optimization Programming Language (OPL) modeling language was used to generate the mathematical models. The solver used in the solution of the instances was the CPLEX 12.10 software. The Visual Basic for Applications (VBA) language was used to process the company data, generate the file to be read by the CPLEX and to analyze the results. The computer used in the processing has an Intel Core i7, 64-bit processor, with 16 GB of RAM. For the solution of the model with real data, due to its high complexity, the processing time limit is set at 8h. With the definition of this broad limit, the objective is to leave the model freer to obtain a quality solution, since the company has this time available to solve their instance. For instances with random data, the limit is set at 4 h.

The instance to be processed by CPLEX is a “.dat” type file, generated in previous processing developed in VBA. This file contains a set with the viable arcs of each instance, that is, arcs that can be used in practice, as they meet the operational restrictions of the company.

To create the set of viable arcs of an instance, first some information about the furnace and the nodes are stored: furnace size; how many nodes will be used (accuracy); which nodes represent sections where movable beams are located; and which nodes represent empty spaces in the oven. Then, unitary arcs are created, which represent space losses in an assignment. A unitary arc is created for each node.

To create the arcs that represent the assignment of items in the furnace, two tests are performed: the initial node of the arc, added to the length of the item, cannot exceed the width of the furnace; along the length of the arc, at least two nodes representing movable beams must be covered. If these two criteria are met, the arc is created, recording the following information: starting node; ending node; represented item; length of this item; its type of bending (parabolic, conventional or straight). This information about each arc enables the structuring of the model based on an arc flow formulation, considering the specific restrictions of the case being studied. After generating the ".dat" file, processing the instance using CPLEX occurs independently.

5.3.1 Real Data

This section deals with actual data obtained from the automotive spring company. Section 5.3.1.1 presents their data and the criteria used. The solution practiced by the company and the solution obtained by the mathematical model are explained in Section 5.3.1.2. In Section 5.3.1.3, the results are analyzed and compared.

5.3.1.1 Data and Criteria

The real data from the company considers one day of production, where 44 types of items are processed in the furnace, made up of 8 parabolic, 15 conventional and 21 straight types of items. The average demand is 51.5 units per item type, totaling 2,266 hardened items in one day. Availability measured at the beginning of the day is 4,071 items, with an average availability per day of 92.5 per item type. The average length of the items is 1,102 mm, and the number of formulas defined by the company is 11. The specifications of speed, thickness limits and temperature of each formula are given in Table 20. The "Time" column represents the time, in minutes, for an item to travel the entire length of the furnace in any given formula. The "Thickness" column shows the range of thickness of the items, in millimeters, included in each formula. Two temperatures ($T1_t$ and $T2_t$) are presented for each formula since, seeking greater precision, the factory defined two measurement points, specifying, for each formula, the temperature to be used at each point.

Table 20 – Formula information (actual data).

Id	Time (min)	Thickness (mm)	$T1_t$ (°C)	$T2_t$ (°C)
1	22	4.8 to 7	880	860
2	20	4.8 to 7	900	860
3	22	6.4 to 9	920	900
4	22	9 to 13	960	920
5	23.5	10 to 13	960	920
6	25	12 to 16	960	930
7	27	15 to 21	960	930
8	30	20 to 28	980	930
9	35	27 to 35	980	940
10	38	34 to 41	980	940
11	41	41 or more	980	940

Due to differences in the shift times of employees in this process, the furnace can be in production for up to 9 hours a day without the need for overtime. Therefore, the time available for production is 540 minutes. As per the limitation of the number of benders after the furnace, each assignment can contain a maximum of 1 parabolic bent item and 3 conventional bent items. As the movable beams take 100 steps to transport an assignment from the beginning to the end of the furnace, it was considered that the furnace holds 100 simultaneous assignments, and that the time for each formula to cross the furnace (shown in Table 20) is the time that the formula takes to produce 100 assignments. It was considered that the setup for any formula consumes 50 assignments, that is, the setup time is half of the traversal time of each formula.

To create an instance with this real data, the value of the margin of each item (mi) was defined in discussion with the production manager, based on information about production costs and sales price of items, to give a value between 1.0 and 2.5. The average margin for this instance is 1.71 per item type. Furthermore, it was considered that each node of the furnace corresponds to 6 cm, that is, the measurements of each section of the furnace (see Figure 14) was approximated by nodes. The total width of the furnace, presented in centimeters in Figure 14 (187 cm) was approximated by 31 nodes. Similarly, the length of the items has also been adapted and the average length is 18.5 nodes.

5.3.1.2 Solution practiced by the company vs. the solution via mathematical model

In its solution, the company uses 1,785 assignments to harden the 2,266 items required in 9h of production. Analyzing this solution with the OF criteria set out in the mathematical model, as for all instances in this study, the value obtained is of 3,845, equal to the sum of the margin obtained with the production of all items. Table 21 shows the OF value of the solution practiced by the company and of the solution obtained by the mathematical model.

Table 21 – OF Value. Company Practice vs Mathematical Model.

OF Comparison	**Objective Function**
Company Practice	3,845.0
Mathematical Model	6,009.5

The company does not measure the percentage lost in the furnace each day. However, regarding their practiced solution, the total length of all assignments (55,335 nodes), there are 1,785 assignments with a length of 31 nodes each. It is also known the total length of produced items (41,911 nodes), obtained by multiplying the length of each item type and the quantity produced. With this, the loss can be calculated, being 24.26% in this case. As shown in Table 22, this calculation is also done for the proposed mathematical solution, which used 1,965 assignments to produce 3,426 items.

Table 22 – Solution losses. Company Practice vs Mathematical Model.

Loss Comparison	**Available Length (nodes)**	**Used Length (nodes)**	**Loss**
Company Practice	55,335	41,911	24.26%
Mathematical Model	60,915	56,708	6.91%

In the practiced solution, 8 formulas were used of the 11 available. Formula 11 was not used because not one type of item processed on this day had the correct thickness for it. Formulas 1 and 10 were avoided through the arrangement of items. Therefore, as shown in Table 23, 102.25 minutes were spent on setups representing 18.9% of the total time (540 minutes). In the mathematical model solution, 6 setups were needed, consuming 75.5 minutes, or 13.98% of the total available time.

Table 23 – Setup time. Company Practice vs Mathematical Model.

Time Comparison	Total Time (min)	Production Time (min)	Setup time (min)	% Setup
Company Practice	540	437.75	102.25	18.94%
Mathematical Model	540	464.5	75.5	13.98%

The total processing time needed to obtain the solution by the mathematical model was 26,804 seconds or 7 hours, 26 minutes and 44 seconds. The CPLEX solver took 21,632 seconds (6 hours and 32 seconds), and the rest of the time (5,172 seconds) was taken to load the data and to create and close CPLEX internal structures. Currently, the company has four employees working in functions related to the hardening furnace. A worker performs assignments by entering items in the furnace, two workers put hardened items in the benders or an oil tank, and one employee is responsible for operating the furnace, setting parameters for the initial heating at the start the day, for the setups and throughout the production, as well as planning all assignments to be made. It is estimated that, out of his 8 hours of daily work, this employee spends 5 hours planning assignments and 3 hours adjusting furnace parameters and performing setups.

5.3.1.3 Additional Analyzes

In this instance, the total availability is 4,071 items, and the production of the solution from the mathematical model is 3,426 items. This means that a good part of the intermediate stock available at the start of the day is used for production over and above the quantity needed (2,266 items). With the implementation of the model, the furnace productivity is increased and there is a tendency for the intermediate stock available at the start of hardening to be consumed.

To deal with this increase in furnace productivity, the company can reduce the time available for the furnace each day, reducing energy expenditure and adjusting the daily production expectation. Another option is to invest in increasing the capacity of previous processes, so that they will continue to be able to maintain a high intermediate stock before the furnace. This option would eventually generate an increase in the total productive capacity of the truck spring sector. The decision between these options is up to company management, based on market data, investment capacity, among other things.

Although the time to obtain the solution is high, this is an expected result, since the real instance is the largest solved in this study, and the instance size has a great impact on computational time. In addition, the waiting time does not prevent the company from running the model daily. At the end of a day, after updating the data relating to demand, inventories and production, the model can be executed. So, at the beginning of the next day, the solution is already available to be put into practice.

To compare the solution practiced by the company and the proposed solution, some factors are analyzed. The overall effect of the factors mentioned below may explain the significant increase in production obtained in the proposed solution (51.2%).

The spring factory produced 2,266 items in 1,785 assignments, which represents 1.27 items per assignment. In the case of the proposed solution, 1.74 items were processed per assignment, producing 3,426 items in 1,965 assignments. This difference is the main reason for the increase in production in the proposed solution and is due to the ability of model to reduce wasted space in the furnace through good combinations of items. This is evident when analyzing the big reduction in wasted space obtained by the proposed solution (71.5%), from 24.26% to 6.91%.

The number of setups performed is also an important factor when comparing the number of items produced in each solution. The practiced solution used 8 setups and the proposed model solution 6 setups. The time spent on setups in the proposed solution was 26.75 minutes less, which represents a reduction of 26.2%. This allows the furnace to spend more time in production and, consequently, allows an increase in the total number of assignments in the proposed solution.

In addition, the use of faster formulas allows more assignments to be made in the same available time. In the solution used, the 1,785 assignments occurred during 437.75 minutes of production, that is, 4.08 assignments were performed per minute. In the proposed solution, there were 1,965 assignments in 464.5 minutes of production, which results in 4.23 assignments per minute. This is a less significant factor but it also helps to explain the increase in the total number of assignments in the proposed solution.

5.3.2 Random Data

In this section, random instances are covered. In Section 5.3.2.1, the criteria are explained, based on the real instance to generate random instances. The results are presented in Section 5.3.2.2.

5.3.2.1 Data Generation

36 groups of instances were defined, which differ through the variation of four parameters: precision; length of items; instance size; and formulas per item. In each group, 5 instances are generated with random data, totaling 180 instances.

Dealing with the precision to be used, two levels of variation were defined. The objective was to simulate the difference between using the precision of one node every 5 cm, and one node every 4 cm, in a 145 cm furnace. Thus, the furnace measures considered in the random instances are presented are in Table 24.

Table 24 – Furnace Information (Random Data).

Section	Support	Length (cm)	Number of nodes (5 cm)	Number of nodes (4 cm)
1	No	21	4	5
2	Yes	9	2	2
3	No	27	5	7
4	Yes	9	2	2
5	No	28	6	7
6	Yes	9	2	2
7	No	12	2	3
8	Yes	9	2	2
9	No	21	4	5
Total		145	29	35

To define the length of the items, three possibilities were established. In instances with small items, the length randomly varies between 350 mm and 600 mm. Medium items range from 500 mm to 750 mm, and for large items the range is from 650 mm to 900 mm. Then, the generated values are converted to nodes according to the precision established for each instance.

Three levels were defined to differentiate the size of the instances. Small instances have 7 item types and 4 formulas, medium instances have 14 item types and 6 formulas, and for large instances there are 21 item types and 8 formulas. Finally, the last parameter is the number of possible formulas for each type of item. The two established limits are shown in Table 25.

Table 25 – Formula levels by item type for each instance size.

		Formulas by Item Type	
Size of Instance	**Small** **7 types of items | 4 formulas**	2	3
	Medium **14 types of items | 6 formulas**	2	4
	Large **21 types of items | 8 formulas**	2	4

For the definition of the values presented in Table 25, the objective is to avoid that the number of formulas per item is 1 and, in the same way, to avoid that the items can be assigned in all the formulas of the instance. As verified in initial tests, in these two cases, the instance is less complex and the analysis is compromised. Therefore, as can be seen in Table 25, two levels of variation were defined for the number of formulas per type of item in each instance. At the lowest level, each type of item can be processed in two formulas. At the highest level, each type of item has four possible formulas, except for small instances, whose number of formulas per item is 3, since 4 is the total number of formulas for these instances. Just as occurs in the real case, the possible formulas for each item in random instances are sequential, i.e. if an item has two possible formulas, and the random choice sets the formulas at 3 for this item, necessarily formula 4 will also be defined.

The other important information for defining the instances was all defined based on real data. The demand for each type of item was randomly defined between 20 and 70. To ensure that there will be availability of items to meet all demand, the minimum value for the availability of an item is the demand for that item multiplied by 1.5. The maximum value that availability can assume is 150. As with the real instance, the margin for each item is randomly set between 1.0 and 2.5. To define the type of item, the same percentage verified in the real instance was kept, 48% straight, 34% conventional and 18% parabolic.

The bender limit was maintained in the random instances, i.e. one parabolic bent item and three conventional bent items per assignment. Items without bends are not limited in this case. The length of the furnace, supporting 100 simultaneous assignments, was also unchanged. The setup time of each formula, in lost assignments, is randomly set between 40 and 60 assignments. The time for a formula to traverse the entire length of the furnace is a random setting between 20 and 35 minutes.

To define the time available for each instance, the minimum time was calculated, if all items are produced in the fastest formula among their possibilities. Furthermore, setup time was defined as 80% of the maximum setup time needed if all formulas are used. Therefore, the time available for each instance was defined as 120% of the minimum production time, plus the time to perform the setups. The purpose of these definitions is to avoid infeasibility due to lack of production time.

To define the number of available assignments in each instance (K), the objective is that the number of available assignments is the smallest that makes it impossible for the model to fully utilize since, in the real case, the limitation is in the production time and not in the number of assignments. Therefore, the setup time and the speed of all formulas of the instance are analyzed. Considering the use of only one formula during the entire production period, the formula that allows the greatest number of assignments is chosen. In practice, it is impossible to obtain a solution that uses a larger number of assignments and, therefore, this is the value to be defined for the K parameter.

5.3.2.2 Analysis of Results

Considering everything explained in Section 5.3.2.1, the 180 random instances were executed respecting the processing time limit of 4 hours for each instance. The results are shown in Tables 26 to 31 below. Table 26 shows the percentage loss obtained on average between the 5 instances in each group, and Table 27 shows the mean values considering the levels of variation of each parameter.

Table 26 – Average loss for each group of instances.

Loss				Instance Size					
				7 types of items 4 formulas		14 types of items 6 formulas		21 types of items 8 formulas	
				Formulas per Item					
				2	3	2	4	2	4
Item Length (mm)	350 – 600	Accuracy (cm)	5	35.5%	25.5%	34.7%	31.2%	34.2%	37.4%
			4	36.1%	37.5%	40.2%	32.2%	31.2%	37.9%
	500 - 750		5	28.6%	40.2%	35.4%	27.4%	32.7%	22.5%
			4	36.0%	32.4%	43.0%	32.6%	40.3%	33.8%
	650 - 900		5	43.0%	40.0%	41.4%	43.8%	36.0%	41.6%
			4	40.6%	43.8%	43.7%	42.9%	43.9%	41.6%

Note that the loss obtained in the random instances is high, in general. This is justified by the reduction in the size of the furnace in random instances, which makes it difficult to design good assignments. In addition, the natural restriction of this problem, with the need for the items to be supported by the beams, also favors an increase in losses. Note that values are naturally higher in instances with large items, as large items make it difficult to generate assignments with low loss. Furthermore, it can also be seen that the definition of more formulas per item reduces the loss, since it increases the possibilities of items to compose the assignments in each formula. The size of the instances does not seem to interfere with the loss.

Table 27 – Average loss of variation for each parameter.

Instance Size		
7 types of items 4 formulas	14 types of items 6 formulas	21 types of items 8 formulas
36.6%	37.4%	36.1%
Item Length (mm)		
350 – 600	**500 – 750**	**650 – 900**
34.5%	33.7%	41.9%
Accuracy (cm)		
5	**4**	
35.1%	38.3%	
Formulas per Item		
2	**3 or 4**	
37.6%	35.8%	

The value of the gap is zero for 178 of the 180 instances, which shows that the quality of the solution is good considering the characteristics of the model and the data of these instances. In the case where the gap is non-zero, it is 0.01% and 0.19% for two instances that have small items, a precision of 1 node every 5 cm and two formulas per item.

Table 28 shows the average computational times for each group of instances and Table 29 shows the average time considering the variation of the parameters.

Table 28 – Average computational time, in seconds, for each group of instances.

Computational Time (s)				Instance Size					
				7 types of items 4 formulas		14 types of items 6 formulas		21 types of items 8 formulas	
				Formulas per Item					
				2	**3**	**2**	**4**	**2**	**4**
Item Length (mm)	**350 – 600**	**Accuracy (cm)**	**5**	580	41	3,228	1,338	2,544	5,779
			4	28	44	653	1,011	3,727	4,053
	500 - 750		**5**	31	56	625	1,473	2,874	11,480
			4	36	48	469	839	4,006	7,269
	650 - 900		**5**	27	54	218	328	1,856	4,090
			4	28	14	202	398	832	2,136

Naturally, the larger the instance, the more the computational time required. In the analysis of this factor, the results of some specific instances significantly altered the average values, although they did not significantly affect the observed trends. Just 2 of the 180 instances reached the 4h of the processing time limit. Instances with larger items use less processing time since the preparation of the assignments becomes less complex in these cases. Also, defining multiple formulas per item tends to increase the complexity of the instance because it increases the possibilities that items compose assignments for each formula.

Table 29 – Average computational time, in seconds, of the variation of each parameter.

Instance Size		
7 types of items 4 formulas	**14 types of items 6 formulas**	**21 types of items 8 formulas**
82	899	4,221
Item Length (mm)		
350 – 600	**500 – 750**	**650 – 900**
1,919	2,434	849
Accuracy (cm)		
5	**4**	
2,035	1,433	
Formulas per Item		
2	**3 or 4**	
1,220	2,247	

Table 30 shows the average percentage of time spent with the completion of setups in each group of instances, and Table 31 shows this same information considering the varying levels of each parameter.

Table 30 – Percentage of time spent on setup in each instance group.

Setup				Instance Size					
				7 types of items 4 formulas		14 types of items 6 formulas		21 types of items 8 formulas	
				Formulas per Item					
				2	3	2	4	2	4
Item Length (mm)	350 – 600	Accuracy (cm)	5	23.3%	26.7%	19.6%	15.9%	18.9%	15.0%
			4	22.8%	22.1%	18.3%	13.8%	18.8%	11.4%
	500 - 750		5	22.0%	15.6%	16.7%	15.6%	14.5%	15.3%
			4	19.1%	18.4%	14.4%	11.8%	13.8%	11.1%
	650 - 900		5	17.2%	14.2%	12.6%	11.6%	13.8%	11.3%
			4	13.5%	11.8%	12.3%	12.1%	12.5%	12.0%

The smaller instances spend proportionately more time on setup. This is because these instances have less flexibility to group items into fewer formulas. This is the same reason why the percentage is higher in instances with two formulas per item. It should be noted, finally, that the instances with large items have the best results in relation to time spent on setup, just as occurs with instances with greater accuracy.

Table 31 – Percentage of time spent in setup with the variation of each parameter.

Instance Size		
7 types of items 4 formulas	14 types of items 6 formulas	21 types of items 8 formulas
18.9%	14.5%	14.0%
Item Length (mm)		
350 – 600	500 – 750	650 – 900
18.9%	15.7%	12.9%
Accuracy (cm)		
5	4	
16.7%	15.0%	
Formulas per Item		
2	3 ou 4	
16.9%	14.8%	

Randomly defining the characteristics of each formula further diversified the quality of the formulas, as the number of assignments lost through setups was not constant. In the real case, the slower formulas will be avoided by the model, since this is the only factor that differentiates between them. In random instances, depending on the data generated, some formulas can be attractive because they are fast but, if the setup time is high, the model can avoid their use as far as possible.

Analyzing the results of the parameters of formulas by item, the tests showed that when the items can be used in several formulas, the computational time increases, since the complexity to reach the optimal solution is also greater. On the other hand, with more flexibility to use items in several formulas, the model can make better assignments, occupying more of the furnace and reducing wasted space. In addition, the model can group items in fewer formulas reducing the need for setups.

It is worth noting that the result obtained with the loss of space in assignments in the real instance is considerably better than the values obtained in the random instances. This is due to the greater diversity of types of items to be assigned in each formula, in both the length of these items and the number of types of items. This diversity provided by the characteristics of the real instance allows the model to make good assignments and so obtain a low loss of space. On the other hand, this same factor is a complicating factor for instance optimization by increasing computational time. Interestingly, the computational time of the real instance was also much higher, even compared to longer random instances.

Analyzing in more detail the processing time in each random instance, it was possible to see that most of the computational time was consumed to reach a feasible solution. In most instances, after reaching a feasible solution, convergence to the optimal solution occurred quickly. This demonstrates the complexity of this problem, in addition to indicating that specific characteristics of random instances accelerate the convergence towards the optimal solution.

5.4 Conclusions

In this study, a problem present in an automotive spring company was studied, seeking to maximize the production of the furnace in the hardening process of truck springs. The problem was approached as an one-dimensional CSP and, considering company specificities, the proposed mathematical model was based on an arc flow formulation.

Tests with real and random data were performed to analyze the performance of the proposed model. In the case of real data, the results showed that the model obtained a better solution than that practiced by the company, with an output of springs 51% higher. The main reason for this was the 71.5% reduction in wasted space in the furnace, reducing empty spaces and assigning more items in the furnace. The reduction in the number of setups performed also contributed to the good result, allowing more time for the furnace to be in production.

The tests with random data allowed the analysis of the effect of varying different parameters, improving knowledge of this still little explored problem, and allowing managers to make better decisions. The computational time is the criterion most sensitive to the variation of parameters in general, mainly to the variation of the instance size, but also the formulas per item and the length of the items. The length of the items is the parameter that most influences the results, strongly affecting the wasted space and the percentage of time spent in setup. Finally, it is important to point out that the solution obtained for random instances is optimal in practically all cases, since, for 178 of the 180 instances, the gap is zero.

6. CONCLUSIONS AND FUTURE PROPOSALS

In this thesis, the production process of an automotive spring factory was studied, aiming at saving resources and increasing productivity. Two studies were performed analyzing the bar cutting process of the company. The focus of the first study was on short term issues, solving an Integrated Lot Sizing and Cutting Stock Problem (ILSCSP) of the type (-/L2/L3/M) (according to Melega *et al.* (2018)) with parallel machines. The second study dealt with medium term in an ILSCSP of the type (L1/L2/L3/M), in which the purchase of objects is a decision variable. A third study was conducted analyzing the assignment of items to a hardening furnace. This chapter carried out a not so common approach, regarding this problem as a CSP of the type (-/L2/-/S) and using a mathematical model based on an arc flow formulation.

Results showed that the models satisfactorily achieved their objective since the solution of instances with real data potentially generated, for the three approaches, a large saving of time and money for the company. In Chapter 3, losses were reduced by 49.7%, saving about 3.3 ton of steel per week. The approach referring to medium term (Chapter 4) reduced losses by 30%, which represents about 75 tons of steel saved in eight months. Furthermore, the proposed solution achieved significant reductions in stock levels of bars, springs and spring bundles, as well as the number of bars purchased and used. In the study from Chapter 5, the model was capable of increasing production of the hardening furnace by 51%, mainly through the reduction (71.5%) of wasted space in the furnace. It is important to state that all solutions were achieved in acceptable computational times, respecting all the specific operational restrictions.

For the performance analysis, the three models were tested in the solving of instances with random data. In total, 540 fictitious instances were solved, demonstrating the influence of several parameters of each problem, and enabling managers to make better decisions. In Chapters 3 and 4, better results, in terms of gap and computational time were found with smaller instances and/or with large items. Instances with small items are more complex and generate worse results, apart from the loss. Additionally, the study in Chapter 4 demonstrates the advantages of considering objects purchases (e_{kt}) as a decision variable. In Chapter 5, similar conclusions were reached about the type of instances that generate better results in relation to computational time and loss. Since the gap value is zero in 178 out of 180 instances, it was not possible to analyze the influence of parameters in this matter. This result demonstrates, above all, the quality of the model in obtaining the optimal solution of these many instances.

Therefore, it is possible to conclude that the objective of this thesis was reached since it demonstrated different uses for the CSP, presenting and analyzing the problem in several contexts in the spring industry, aiding the company in improving decision making and saving of resources. Moreover, contribution has been made to the literature in the study of yet underexplored approaches, along with the proposition of unprecedented methodology in this context. The relation between the CSP and other classic problems was also studied, since an approach integrated with LSP was applied on Chapters 3 and 4 and the mathematical model of the article in Chapter 5 is based on an arc flow formulation.

As future research related to Chapter 3, improvement on the solution approaches might be interesting, such as stabilization techniques, reformulations and heuristic procedures to find an integer solution. It is also possible to analyze the performance of the approach used in other companies, whose production process is similar. Regarding the application in some other contexts, for a satisfactory result, it might be necessary to include in the model the setup times of the cutting machines. In addition, the production capacity of the cutting process can be considered as a parameter, enabling an in-depth analysis of possible advantages of increasing or reducing capacity, since more capacity would lead to more idleness, but more freedom to cut on the preferred date.

Considering Chapter 4, with the objective of bringing this work closer to practical issues, different elements related to the bar purchase variable e_{kt} should be considered: for example, the cost of ordering, delivery time, different unit costs depending on the purchase quantity, prices of different suppliers, among others. For the spring bundles, a production limit was not considered, as this does not make sense in the studied company. However, it could be easily inserted in the model with a capacity restriction to limit the value of the variable y_{pt} in each period t.

As proposals for future studies related to Chapter 5, the author suggests that computational time should be further analyzed, since it is sensitive to the increase in instance size, in addition to having a specific behavior, taking a long time to find a feasible solution but then converging quickly to the optimal solution. Changes in the mathematical model reducing the number of nodes and edges of the network, seeking a simplified solution without loss of quality can be assessed. Alternatively, heuristic methods can be developed to reach good solutions in a shorter time and also to avoid the purchase of commercial software by the company.

Although the optimal solution was obtained in almost all random instances of Chapter 5, the absolute value of the space loss was generally high. Therefore, it is suggested that a more detailed study be carried out, in search of the main characteristics of the instances, which account for the increase in loss of space in the furnace. Finally, an interesting line of study is the analysis of this problem considering multiple periods. The solution of a one-dimensional multiperiod CSP, already studied in different contexts, can produce better results than approaches looking at an isolated period (Poldi and De Araujo, 2016). However, it is important to carefully assess the impact of the likely increase in computational time. One possible result for the case studied here could be the decision that, in one day of production, the company produced only items of small thickness, keeping the furnace cooler and making fewer setups. The next day, the furnace could start hotter, focusing on the production of thicker items. This might produce better results than those analyzing each day separately.

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