UNIVERSIDADE TECNOLÓGICA FEDERAL DO PARANÁ

CALEQUELA JOÃO TOMÉ MANUEL

ALGORITHMS ASSISTANCE OF THE CAR MANEUVERS DURING PARALLEL AND PERPENDICULAR PARKING

THESIS

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CALEQUELA JOÃO TOMÉ MANUEL

ALGORITHMS ASSISTANCE OF THE CAR MANEUVERS DURING PARALLEL AND PERPENDICULAR PARKING

Algoritmos de assistência as manobras do carro durante o estacionamento paralelo e perpendicular

Thesis presented as a requirement to obtain the title of
PhD in Industrial Engineering at the Universidade
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Co-Advisor: Prof. Dr. Max Mauro Dias Santos.

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CALEQUELA JOAO TOME MANUEL

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Trabalho de pesquisa de doutorado apresentado como requisito para obtenção do título de Doutor Em Engenharia De Produção da Universidade Tecnológica Federal do Paraná (UTFPR). Área de concentração: Gestão Industrial.

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Dedico este trabalho à minha esposa
Fabíola R. G. Manuel e ao meu filho Raí
Bless Manuel, pelos momentos de
ausência.

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"Imagination is more important than knowledge.
Knowledge is limited, while imagination cuddle the
entire world, spurring progress, and giving rise to
evolution."

- Albert Einstein -

(EXAME. 20 sentences by Albert Einstein to understand life, science and art. **Science.** 29 may. 2021)

RESUMO

Um projeto veicular deve considerar e garantir a segurança do motorista e de terceiros. Destaca-se como fatores críticos o controle de ação dos pedais do carro (embreagem, freio, acelerador) e o controle visual dos pontos cegos (frontal, traseira, esquerda, direta e laterais) que devem ser controlados simultaneamente durante a execução da manobra de estacionamento de um carro. O desenvolvimento de um produto na indústria 4.0 consiste na sequência de etapas/processos utilizados como avaliações métricas em relação ao tempo, economia, aplicação e inovação. A sequência positiva de simulações/testes computacionais no modelo *Software-in-the-Loop (SIL)* reduz/elimina o risco de obter uma validação não bem-sucedida no modelo *Hardware-in-the-Loop (HIL)*, visando a economia de componentes ou dispositivos eletrônicos. A redução de esforço humano, tempo menor de manobra durante o estacionamento automático do carro, e a garantia de segurança que proporciona conforto torna-se um atrativo extra para os consumidores adquirirem o produto; por outro lado um grande desafio para indústria 4.0/automobilística que deve tornar o valor da compra do produto (sistema de auxílio a manobra automática para carros) acessível. No entanto, esta tese de doutorado, propõe e válida como solução o desenvolvimento de um pacote de software integrado por 2 algoritmos de auxílio a manobra do estacionamento automático para carros, destacados como: Algoritmos paralelo e perpendicular. O desenvolvimento dos algoritmos foi baseado na linguagem de programação (C) e representado em (.m) por ser um tipo de linguagem de fácil implementação, pela qual assegura o funcionamento correto em grande parte dos módulos ou barramentos de comunicações dos carros elétricos, híbridos e autônomos. Os testes foram validados no modelo computacional *Software-in-the-Loop (SIL)* representados em 2D com auxílio do software Matlab/Simulink. Portanto, os resultados positivos atestam a veracidade dos algoritmos baseado nas equações matemáticas/geométricas projetadas para controlar as manobras do carro durante o estacionamento.

Palavras-chave: estacionamento automático; algoritmos de manobras; validação de produto; simulações computacionais; produto da indústria 4.0; sistemas embarcados.

ABSTRACT

A vehicular project must consider and guarantee the safety of the driver and third parties. As critical factors are the control of the action of the car's pedals (clutch, brake, accelerator) and the visual control of the blind spots (front, rear, left, right and laterals) that must be controlled simultaneously during the execution of a car parking maneuver. The development of a product in Industry 4.0 consists of the sequence of steps/processes used as metric evaluations in relation to time, economy, application and innovation. The positive sequence of computational simulations/tests in the *Software-in-the-Loop (SIL)* model reduces/eliminates the risk of obtaining an unsuccessful validation in the *Hardware-in-the-Loop (HIL)* model, aiming at the economy of electronic components or devices. The reduction of human effort, shorter maneuvering time during automatic parking of the car, and the guarantee of safety that provides comfort becomes an extra attraction for consumers to purchase the product; on the other hand a great challenge for the 4.0/automotive industry that must make the purchase price of the product (automatic maneuver assistance system for cars) accessible. However, this doctoral thesis, proposes and valid as solution the development of a software package integrated by 2 algorithms of maneuver of the automatic parking for cars, highlighted as: Parallel algorithms and perpendicular. The development of the algorithms was based on the (C) programming language and represented in (.m) as it is a type of easy-to-implement language, which ensures the correct functioning of most modules or communications buses of electric, hybrid, and autonomous cars. The tests were validated in the computational model *Software-in-the-Loop (SIL)* represented in 2D with help of Matlab/Simulink software. Therefore, the positive results attest to the veracity of the algorithms based on the mathematical/geometric equations designed to control the maneuvers of the car during parking.

Keywords: automatic parking; maneuver algorithms; product validation; computer simulations; industry 4.0 product; embedded systems.

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LIST OF SYMBOLS AND ABBREVIATIONS

UTFPR	Universidade Tecnológica Federal do Paraná
PG	Ponta Grossa
OpenCV	Open-Source Computer Vision Library
LiDAR	Light Detection and Ranging
Tanh	Tangent hyperbolic
Cycab	Battery-Powered Electric Vehicle
VR	Virtual Reality
2D	Two Dimensions
3D	Three Dimensions
SEVA	Autonomous Vehicle Parking System
SIL	*Software-in-the-Loop*
HIL	*Hardware-in-the-Loop*
EP	Engenharia de Produção
CAPES	Coordenação de Aperfeiçoamento de Pessoal de Nível Superior
ANN	Artificial Neural Networks
IDE	Integrated Development Environment
JacintoNet	Friendly Convolutional Neural Network
MPC	Model Predictive Control
4WS	Four-Wheel Steering
FWS	Front-Wheel Steering
ADAS	Advanced Driver Assistance System
PID	Proportional Integral Derivative
CAN	Controller Area Network
ICC	Instantaneous Center of Curvature
Obs	Observations
PPGEP-PG	Programa de Pós-Graduação em Engenharia de Produção-Ponta Grossa
in	inch
mm	millimeter
cm	centimeter
m	meter

SUMMARY

1 INTRODUCTION

In the automotive industry the technological evolution is referenced by sensor systems, according to (BHOI; KHILAR, 2014). Nicklaus Otto in 1876 invented the four-stroke internal combustion engine (FORRESTER, 2020). The German designer and engineer Karl Benz by profession improved the invention implementing to car; the increase in the popularity of vehicles was consecrated in 1904 when engineers managed to solve the problems of noise, odors and vibrations that originated from the vehicle's engine (BHARADWAJ, 2018).

Henry Ford in 1896 (ROYSTON, 2015) produced the first T-model car designed with the purpose of affordable price and greater purchase, highlighting the production of cars that was carried out in less time where in 1914 it implemented a production system that allowed workers to assemble a car in 30 minutes (CURCIO, 2013).

For automobiles, industrial innovation standards are complex because it is a large product (PAUWELS *et al.,* 2004) it does not have the same ease of developing digital devices or software that are operated by logic and electronic systems without significant weights. The automobile industry follows in response as technological/industrial innovation; the reduction/elimination of carbon emissions from the internal combustion engine (REITZ *et al.,* 2020; WALMSLEY *et al.,* 2015; YU *et al.,* 2018) vehicle weight reduction, vehicle operation with one energy source totally ecological, avoid accidents or incidents between vehicles and pedestrians (FUJIMOTO, 2014).

Safety operational and lower purchase acquisition cost so that the demand has the balance of the offer of the products (CHRISTOPHER, 2010; YIN; STECKE; LI, 2018; CHOPRA; SODHI, 2014). As a solution, the analytical structure that involves past and present scenarios and dynamic future estimation of the automotive industry is represented graphically or by results to evaluate the quantity and quality of this structure that can be applied in other industries outside the automotive context (CUMINGS, 1984).

The technology that establishes communication and integrates other components of a vehicle (CHANG *et al.,* 2017) is increasingly gaining importance and prominence in the vision of consumers so that they are connected, informed and safe during urban and rural traffic. The urban mobility infrastructure consisting of cameras and sensors helps in connectivity with vehicles (KAIWARTYA *et al.,* 2016) by which it allows extracting information about environmental and traffic conditions (GUERRERO-IBÁÑEZ; ZEADALLY; CONTRERAS-CASTILLO, 2018). The set of sensors implemented in a vehicle characterize the (ITS) Intelligent Transport System (SCHIEBEN *et al.,* 2019).

1.1 Research problem

When parking a vehicle in a parallel or perpendicular way, it requires the driver to have adequate space and consider the blind spots during the maneuver, in addition to controlling the steering wheel, mirrors, the pedal set (accelerator, brake, clutch) and ensuring the safety of all vehicle occupants and third parties; to avoid accidents/incidents according to Photography 1.

Photography 1 - Accident/incident when parking the car.

Source: NBC 5 (2014).

Due to the complexity of the maneuver during parking, which parameterization techniques (decision process) are adopted?

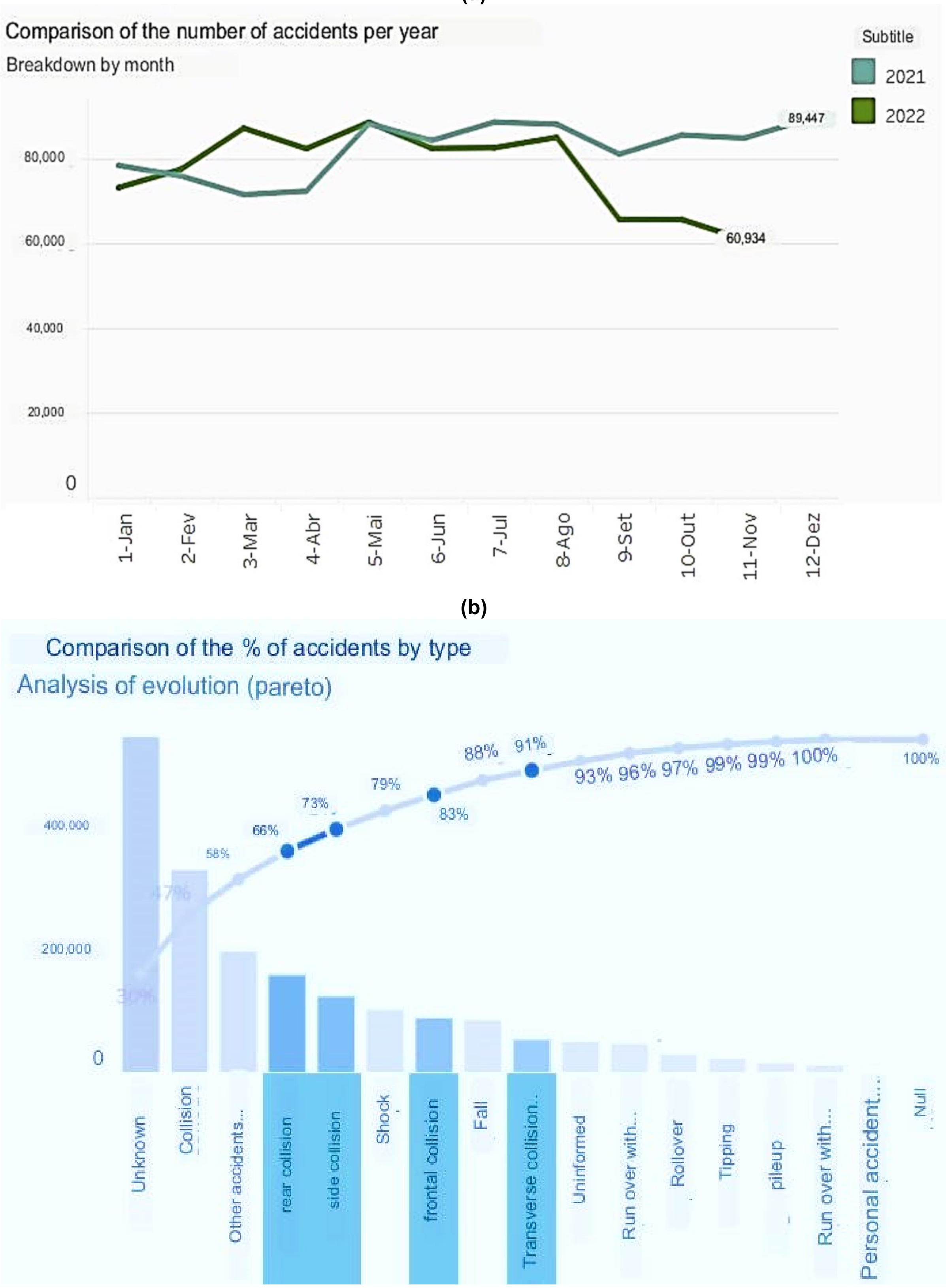
How can the maneuver assistance system be low cost and robust to parametric variations?

Because it is a product (maneuver assist system) in the cycle of the automobile industries, which operational sequence is inserted in the projects to guarantee the non-waste of resources and the correct validation of the product?

The statistical graphs made available by the (MINISTRY OF INFRASTRUCTURE, 2023) of the Brasil between the years 2021 and 2022, are represented in Graphic 1.

Graphic 1 - Statistics of traffic accidents/incidents in Brasil between the years 2021 and 2022: (a) number of accidents/incidents between the years 2021 to 2022, (b) main causes of accidents/incidents.

(a)



Source: Ministry of Infrastructure (2023).

Highlights that rear, lateral (side), frontal, and transverse collisions are the main causes of traffic incidents/accidents in Brasil.

1.2 Research delimitations

This research is delimited in the concept and development of parallel and perpendicular algorithms, used to assist the maneuvers of front-steering vehicles, classified as an automatic parking system.

1.3 Justification

The main justifications this doctoral thesis is; reduction of human effort, avoid accidents or incidents during parking the car, encourage and accelerate the production of cars with automatic parking system already integrated and possible integration of the algorithms as a Simulink toolbox.

The algorithms used for the automatic parking system of a vehicle, after validation *Software-in-the-Loop (SIL)* and *Hardware-in-the-Loop (HIL)* are considered as products in the automotive industries. *Software-in-the-Loop (SIL)* validation enables cost and time savings during testing on automotive components. Obtaining an automatic parking system in a parallel and perpendicular way is an extra attraction for consumers/customers, considering the reduction of human effort and the guarantee of safety during vehicle parking.

This thesis includes directly/indirectly the areas* and subareas of knowledge related to Production Engineering according to ABEPRO (Associação Brasileira de Engenharia de Produção), highlighting the areas of:

- Operations Engineering and Production Processes;
    - Subareas (planning, programming, production control and methods engineering).

- Operational Research;
    - Subareas (modeling, simulation and optimization; mathematical programming and computational intelligence).

- Quality Engineering;
    - Subarea (reliability of processes and products).

Product engineering;
  - Subareas (product development process; product planning and design).

* Economic engineering;
  - Subarea (cost management).

Furthermore, this thesis has correlation with SDG (Sustainable Development Goals) characterized by the 9ª objective (Industry, Innovation, and Infrastructure).

This thesis aggregates the use of linear and non-linear dynamics study techniques and control techniques during the manufacturing process of a product in industry 4.0 (automotive industry) by which it presents a software package represented by algorithms to assist the maneuver for the automatic parking system.

1.4 General objective

Development, representation, and simulation of an automatic parking software package for small or large size vehicles, integrated by parallel and perpendicular algorithms; that can be implemented in different types of cars (Electric, hybrid and Autonomous);

Model and simulate the 4-wheeled car based on longitudinal and lateral movements during the parking maneuver;

Highlight the importance of geometry/mathematics in the construction of an algorithm to assist the driver during the maneuver of the car;

Make the developed algorithms available in a clear way to facilitate the industries during product validation, allowing a balanced or lower value for the final customer;

Ensuring the safety and comfort of all car occupants and third parties during automatic parking based on the maneuver algorithms;

The development of this project will contribute to the development of research on emerging themes for automotive technological development.

1.5 Specific objectives

I. Develop parallel and perpendicular algorithms for the automatic vehicle parking system;

II. Validate the kinematic and dynamic modeling of the front-steering vehicle from geometric/mathematical equations, classified as *Software-in-the Loop (SIL)* validation;

III. Simulations of algorithms in graphical interface (2D);

IV. Model and run the algorithm successfully for further validation *Hardware-in-the-Loop* (Prototypes or real cars);

V. Assign data and initial configuration to the steering system of a basic, electric, autonomous, hybrid or intelligent car;

VI. Accelerate the process of driver assistance system implementation in the automotive industry.

1.6 Research structure

This Doctoral Thesis consists of the division of chapters represented in the following structure:

VII. Chapter 2 presents the bibliographic survey of some research/projects whose theme is related to the maneuver assist the car during automatic parking;

VIII. Chapter 3 describes the principles of kinematics and dynamics modeling added to the 4-wheels car and how a kinematic variable can influence the dynamic performance;

IX. Chapter 4 directs classical geometric concepts on the maneuver control algorithms for parking cars;

X. Chapter 5 refers to the main geometric/mathematical equations used in computer simulations to validate the maneuver assist algorithms with the objective of automatically parking cars in a parallel and perpendicular form; the results of the computer simulations, and later the discussion of the best performance of the algorithm;

XI. Chapter 6 brings the final considerations of this Doctoral Thesis; future considerations on general or specific points for continuation this scientific research, and the curricular summary of the entire trajectory during the Doctoral course up to the date of the final presentation of this thesis.

Is describes the theoretical references of this doctoral thesis in the next chapter (2).

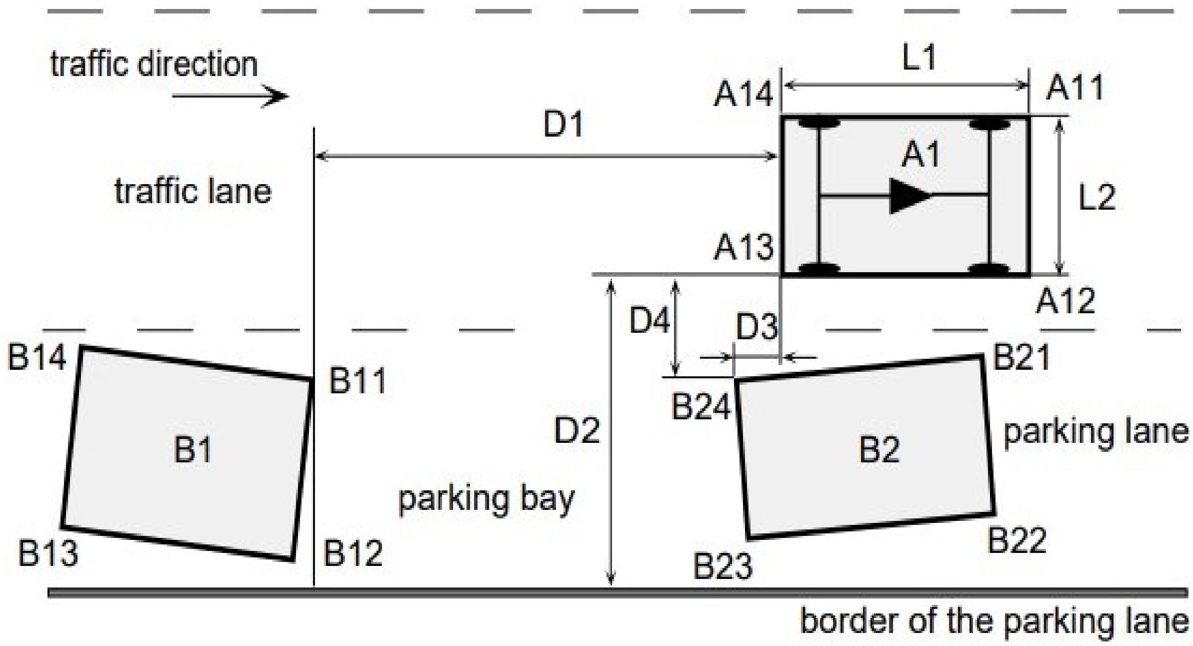
2 THEORETICAL REFERENCE

In this chapter (2), some works/research based on algorithms for controlling maneuvers during automatic parking of a car, presented in the state of the art, are discussed (subchapter 2.1). The problem of developing automatic driving and parking assistance systems has aroused the interest of many academic researchers and the automobile industry. For the theoretical reference in relation to the research topic and the research problem (ERRAJI, 2018) highlight that the parking system of a vehicle parallel and perpendicular one of the great difficulties is when there is not much information about the vehicle's position in relation to the space where you want to park. Blind spots in vehicles during parking have been one of the biggest criteria for accidents or incidents for both vehicle occupants and third parties (MARTÍNEZ-DÍAZ; SORIGUERA; PÉREZ, 2019).

2.1 Theoretical and practical evolution of algorithms

The authors Paromtchik and Laugier (1996) described an automatic parking problem for a non-holonomic vehicle, the resulting algorithm for the developed parallel parking maneuver was based on the geometric principles shown in Figure 1.

Figure 1 - Geometric principles for parking the car.



Source: Paromtchik *et al* (1996).

Implemented in the LIGIER autonomous electric vehicle and was validated with success.

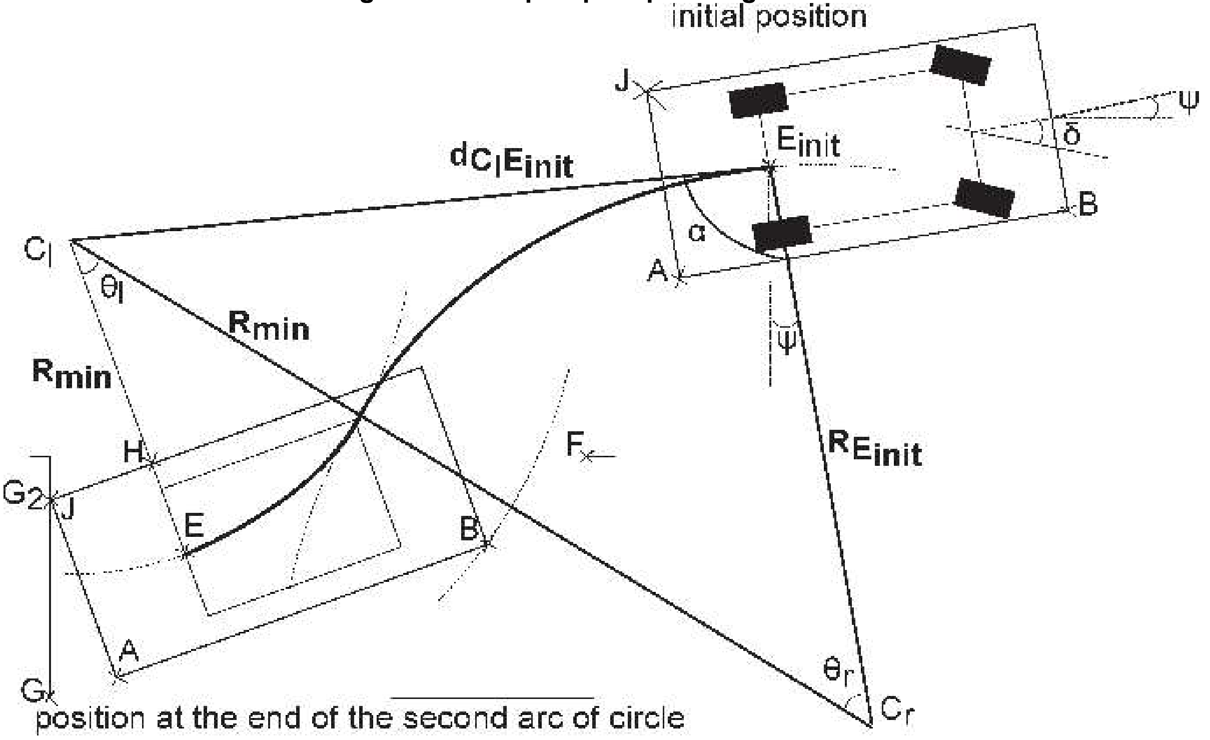
The results presented in Heinen *et al.,* (2007) and Wolf *et al.,* (2009) described in an organized way the main concepts involved in machine learning algorithms, the SEVA project - Autonomous Vehicle Parking System consists in the development of an intelligent control system (use of Artificial Neural Networks - ANN) capable of controlling the vehicle during the maneuver to park in the intended parallel vacancy.

Gupta and Divekar (2010) developed a simple and accurate algorithm for the automatic parallel parking system based on Ackerman's steering configuration, where the system uses sonar sensors and wheel encoders for its perception. De Oliveira Andrade, Hernandes and Becker (2012) presented the design of a simulation environment in language (C#) with Visual (C#) 2008 IDE to validate the algorithm for parking a vehicle in parallel based on data from front laser sensors and rear.

Razinkova, Cho and Jeon (2012) validated an intelligent automatic parking system where data is introduced to automatically generate the parking trajectory using fuzzy logic (multiple logical values, in which the truth values of the variables can be any real number between 0 and 1): As a result, the high robustness and variation of the parking parameters consider the velocity of the car.

Neves, Delamaro and Masiero (2014) developed a structural support environment for autonomous vehicle testing, added to a meta-model which stores information about test scenarios, logs, heuristic program versions for selected discriminative segments of execution logs. The theoretical and practical research developed by Vorobieva *et al.,* (2015) presented the automatic parallel parking system for a passenger vehicle, where the path planning method consists of two parts, first; geometry-based kinematic model of the vehicle, second; the path is transformed into a continuous curvature; according to Figure 2.

Figure 2 - Two-part path planning method.



Source: Vorobieva *et al* (2015).

Petrov and Nashashib (2015) For the problem of the vehicle parking system in reverse (perpendicular) a saturated nonlinear controller (tanh type) with feedback direction for straight line tracking was designed, as a result the controller was able to quickly achieve the direction avoiding vibrations and the first tests on the automatic Cycab vehicle confirmed the validation of the proposed controller.

Muoh *et al.,* (2017) recommended a virtual reality simulator that can be used to improve parking skills since its design, implementation and an experiment related to their case study, this application was validated using- a headset, VR glasses and a Unity 3D game engine to observe the effectiveness of simulators during driver training.

Jeanneret *et al.,* (2017) developed tools to connect several real and virtual devices simultaneously, such as the driver, the racing player's joystick, the real engine, the virtual powertrain and the virtual environment of the driver in order to make the HIL simulation more realistic based on direction mesh human, where the primary results were positive and ensured the modularity of the tool.

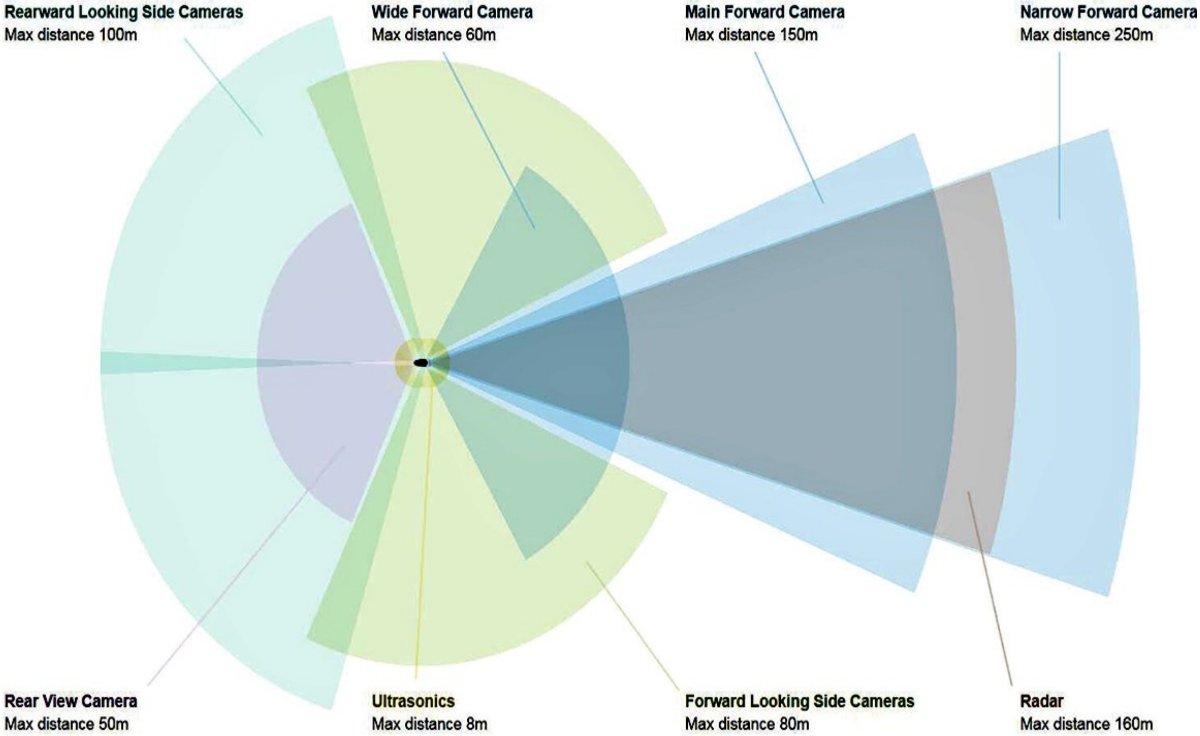
The parking control algorithms implemented by the Tesla Model S industry, features an autopilot system, an autonomous system that steers the car without driver intervention in addition to the automatic parking system long as the destination is informed, the developed car model is shown in Photography 2.

Photography 2 - Tesla Model S car.

Source: Tesla (2016).

The Tesla Model S has many sensors, being; 8 long range video cameras distributed around the car, 12 ultrasonic sensors to detect objects up to 8m away, 1 radar sensor installed in the front part of the car allowing increased 360° visibility up to 250m away, as shown in Graphic 2.

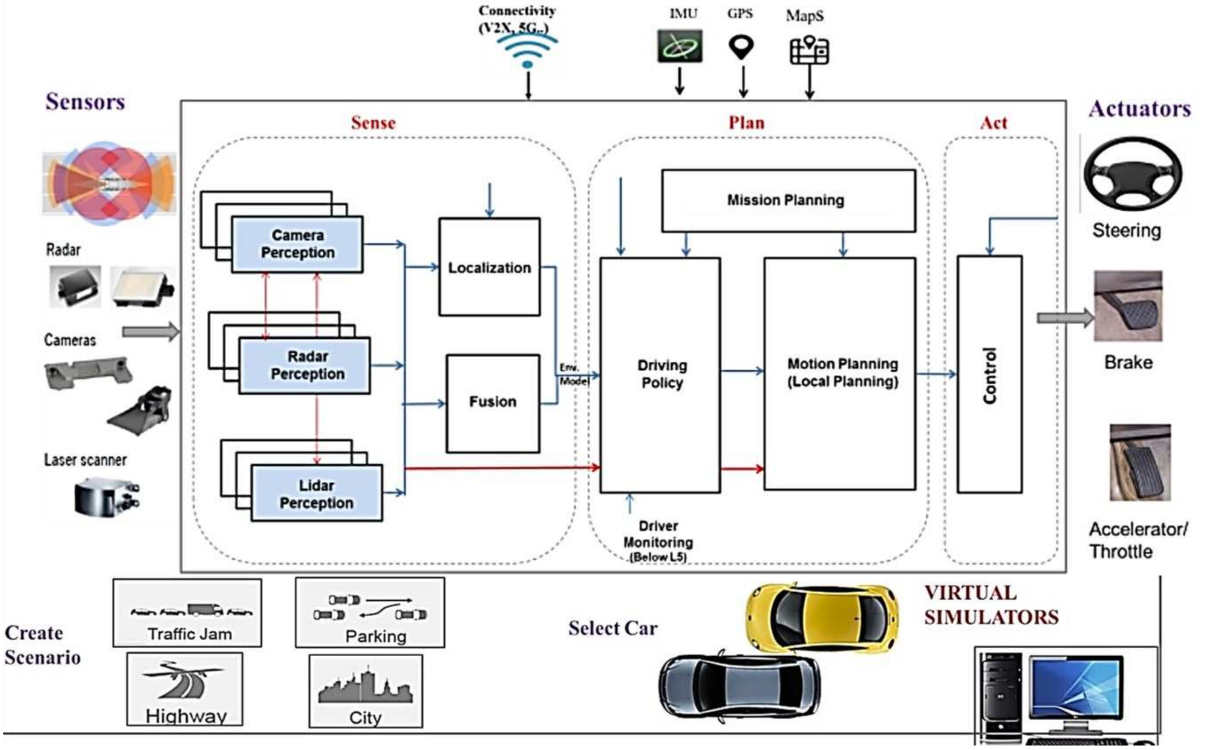
Graphic 2 - Set of sensors implemented in the Tesla Model S car.



Source: Tesla (2016).

Viswanath *et al.,* (2018) the authors describe/present the main requirements when using virtual simulation platforms in the creation of software and algorithms for autonomous driving, as shown in Figure 3.

Figure 3 - Algorithms integrated into the virtual simulation platform.



Source: Adapted from Viswanath *et al* (2018).

In detail, the steps for the construction of a virtual simulation platform aiming at a solution for the development and testing of software were presented, two steps of software development and algorithms for autonomous driving were proposed, the results presented met some requirements that allow margins for new surveys, they need to be adjusted in future surveys to meet all requirements.

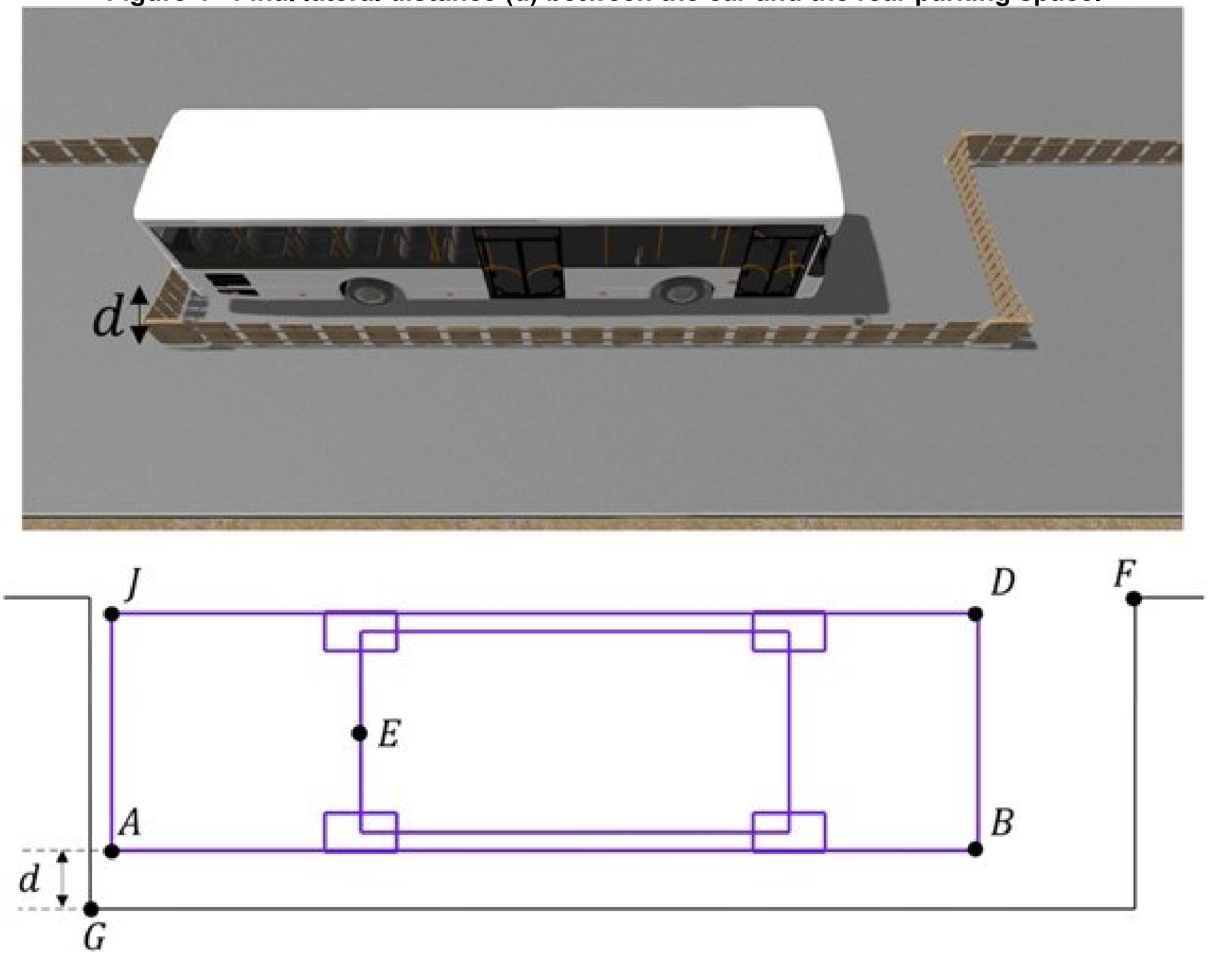
Viswanath *et al.,* (2018) proposed the design of an embedded friendly convolutional neural network (JacintoNet) to demonstrate self-driving in a virtual, where the results are displayed on an embedded platform.

Sedighi; Nguyen and Kuhnert (2019) and Kupresak (2020) consolidated an algorithm consisting of a set of predefined parking maneuvers capable of processing ten parking shapes per second, the complete algorithm is written in Python programming language using the OpenCV image processing library.

Riva (2020) proposed an automatic parking strategy in the perpendicular form of a three-wheel scooter, the algorithm is based on measurements of the surrounding environment obtained through a low-cost LiDAR sensor and takes advantage of vehicle odometry reconstructed through two encoders.

Metin and Sezer (2021) recommend a solution for minimum final side distance (d) to parking space constraints, where the minimum value of (d) depends on the parking space and car parameters, in particular cars with longer length such as buses, according to Figure 4.

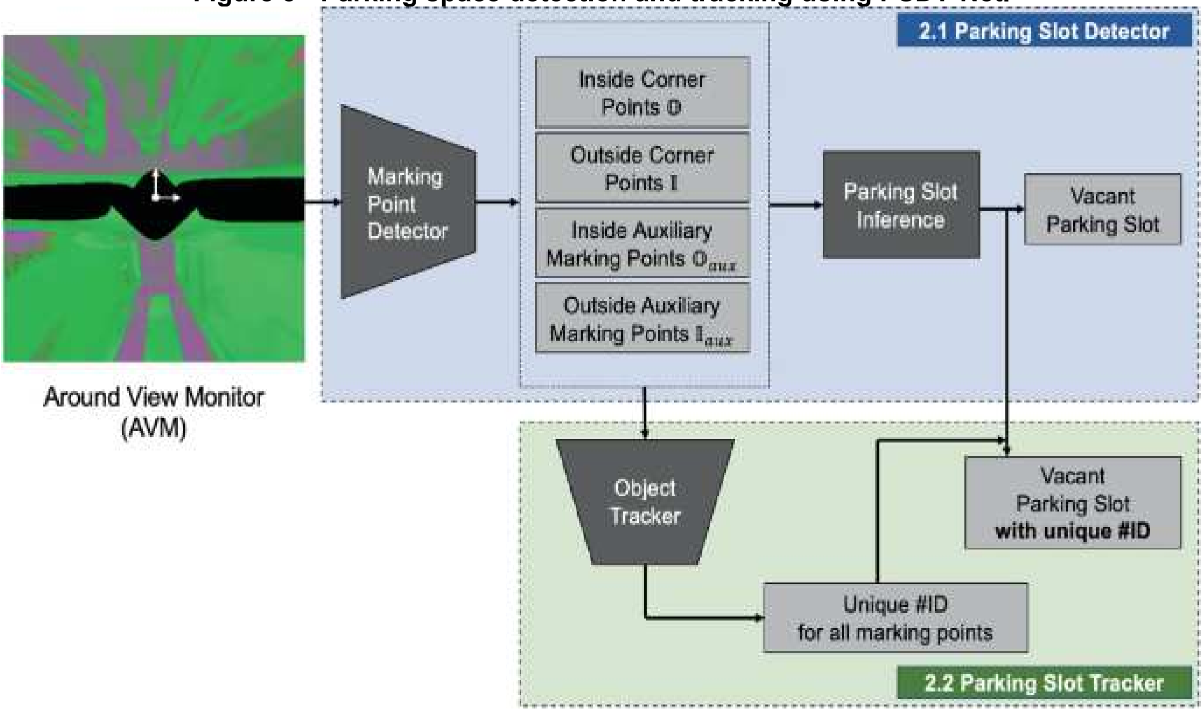
Figure 4 - Final lateral distance (d) between the car and the rear parking space.



Source: Metin and Sezer (2021).

Park; Ahn and Park (2022) have defined a Deep Learning based parking space detection and tracking algorithm, defined by segmentation and easy blind spot recognition to ensure the highest accuracy in choosing parking spaces, named as (PSDT-Net) Parking Slot Detection Tracker, as shown in Figure 5.

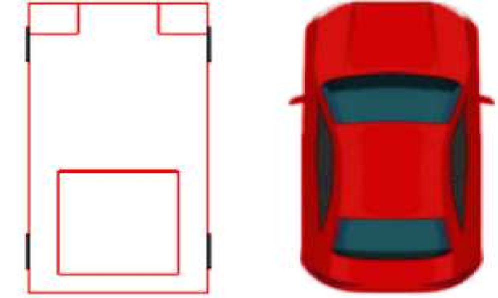
Figure 5 - Parking space detection and tracking using PSDT-Net.



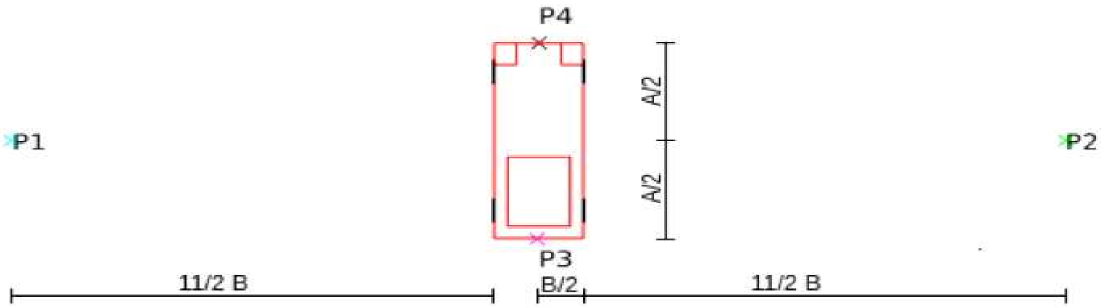
Source: Park; Ahn and Park (2022).

Vieira; Argento and Revoredo (2022) validated an algorithm for path planning for car-like mobile robots (CLMR) with the ability to move from an initial to a final position, the environment in which the CLMR navigates is used a map with geometric primitives of easy modeling in computational terms, and to face the obstacles present in the navigation environment four circular sensing zones were defined, centered on four named reference points (P1, P2, P3, P4), according to Figure 6.

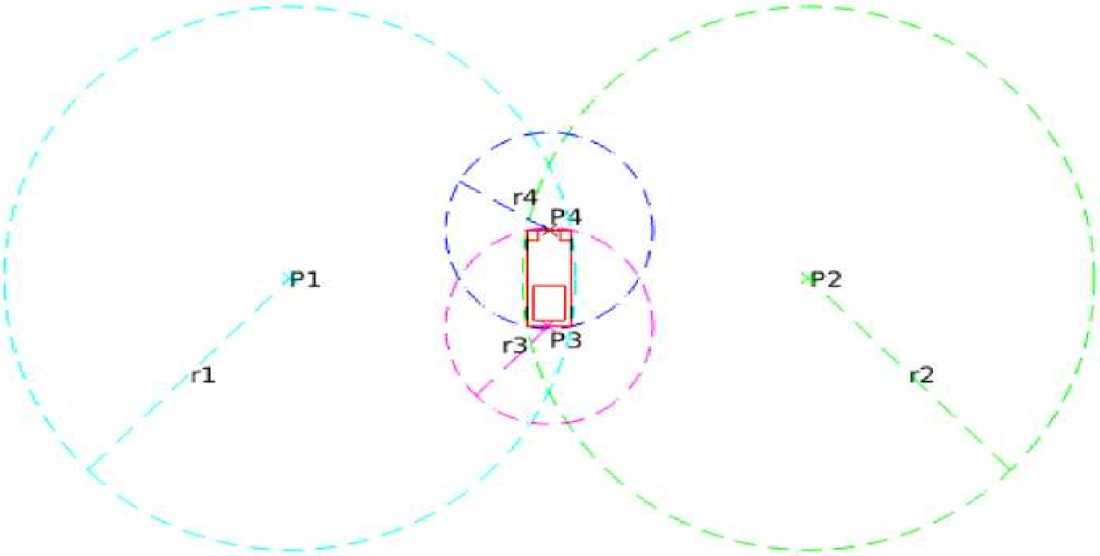
Figure 6 - Obstacle detection zones while parking.



(a) Representation of the vehicle



(b) Four reference points of detection



(c) Detection zones

Source: Vieira *et al* (2022).

In the next chapter (3) the kinematic and dynamic modeling of a car is described.

3 KINEMATICS AND DYNAMICS FOR MODELING A CAR

This chapter (3) describes the kinematic and dynamic modeling of a 4-wheels vehicle, represented by the mathematical equations and the geometry associated with its main dimensions. The results of the computational simulations; proved that changing one kinematic variable directly influences the dynamics of the vehicular system. This chapter was/is the fundamental basis for the development of the automatic parking algorithms, the study of the main kinematic and dynamic variables of the car makes it possible to calculate the initial and final radius of each maneuver.

3.1 Introduction

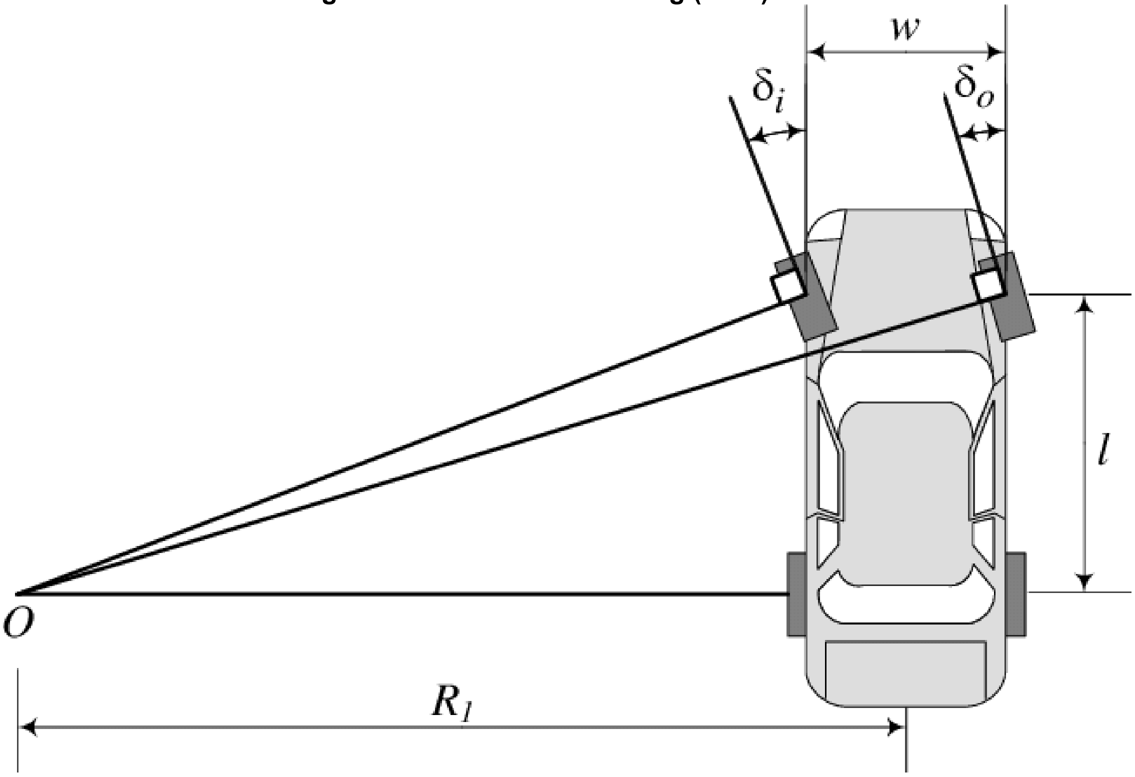
The concept of car steering performance depends on several factors, in highlight the car's total mass, center of gravity, force/traction, velocity and acceleration; according to the designated design when the car moves over the earth's surface. The different vehicular steering designs are often classified as: FWS (Front-Wheel Steering) and 4WS (Four-Wheel Steering). FWS cars are marketed for urban use or for less complex tasks requiring less wheel traction on the two front wheels (TAN; LIU; XIONG, 2022). While 4WS (Four-Wheel Steering) cars are marketed for rural/sports use or for more complex tasks requiring more wheel traction on the four wheels, in highlight recalibrated suspension, firmer shock absorbers, and more engine power (MOON *et al.,* 2022) and (NAGAI; TSUCHIYA, 2022).

Algorithms employed in ADAS (Advanced Driver Assistance System) systems perform their tasks greater accurately when the car's physical dimensions/information is recorded/programmed correctly (CHELBI; GINGRAS; SAUVAGEAU, 2020). To ensure the stability of the car during the operational phase the control system of the car must be triggered automatically (CHU *et al.,* 2018) the intelligent and autonomous cars are apt to implement different types of controllers such as; PID (Proportional Integral Derivative), MPC (Model Predictive Control) and others(a) that correct the movement of the vehicle quickly and with better precision and accuracy (SAMUEL, 2019) even when the vehicular system is considered robust.

3.2 Kinematic modeling of the FWS

Front-Wheel Steering (FWS) The Ackerman geometry can be presented through a simplified model of a vehicle with FWS steering system (RAJA *et al.,* 2021; GUPTA *et al.,* 2019) and that makes a left turn at low velocity; the slip of the tires due to the change of direction is not considered; according to Figure 7.

Figure 7 - Front wheel steering (FWS) model.



Source: Jazar (2017).

To obtain geometric/mathematical equations, it is necessary to know the variables shown in Front wheel steering-FWS model (Figure 7), as described in Frame 1.

Frame 1 - FWS model variables.

Variables	Descriptions
δ_i	Inner wheel steering angle
δ_o	External wheel steering angle
w	Gauge or distance between wheels on the same axle
l	Length between the axis
o	Rotation center
R_1	Distance from center of rotation (o) to center of gauge (w)

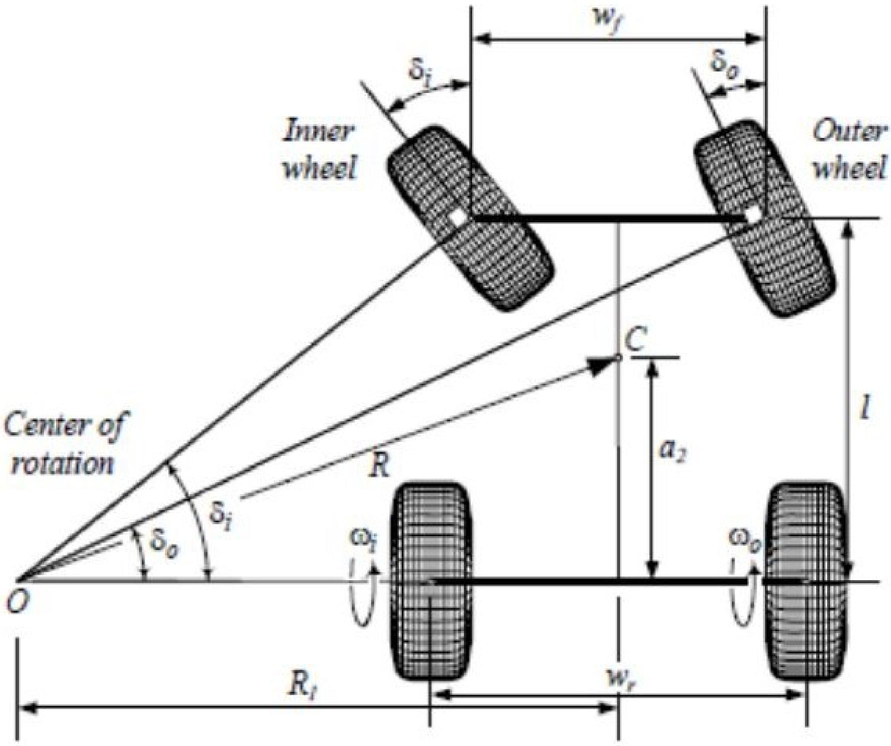
Source: Own author (2023).

In this case, the Ackerman geometry is used for the steering alignment of the two front wheels represented by equation 3.1:

$$\cot \delta_0 - \cot \delta_i = \frac{w}{l} \ . \tag{3.1}$$

Usually in the light vehicles, the front and rear track gauges are the same size. Due to the complexity of movement during the travel of racing vehicles, usually the rear track gauge is larger than the front. However, the Ackerman geometry achieves geometric equilibrium of the vehicle model, as shown in Figure 8.

Figure 8 - Model of a vehicle FWS based on Ackerman geometry.



Source: Bari *et al* (2014).

After the proper geometric representation of the FWS vehicle model associated with Ackerman geometry it is possible to calculate the angular velocity (r), highlighted by equation 3.2:

$$r = \frac{Rw.\omega_o}{R_1 + \frac{w_r}{2}} = \frac{Rw.\omega_i}{R_1 - \frac{w_r}{2}}, \tag{3.2}$$

the variables presented in the model of a vehicle FWS based on Ackerman geometry (Figure 8) are adequately described by Frame 2.

Frame 2 - FWS model variables based on Ackerman geometry.

Variables	Descriptions
r	Vehicle angular velocity
R_w	Tire radius
ω_o	Angular velocity of external wheels
ω_i	Angular velocity of internal wheels
f $subscribed$	Front
r $subscribed$	Rear
o $subscribed$	External
i $subscribed$	Internal

Source: Own author (2023).

The steering of the front wheels is based on the inside and outside steering angles.

The intern steering angle is calculated from equation 3.3:

$$\delta_i = \tan^{-1}\frac{2l(\omega_o+\omega_i)}{w_f(\omega_o-\omega_i)+w_r(\omega_o+\omega_i)}\,, \tag{3.3}$$

and the extern steering angle is calculated using equation 3.4:

$$\delta_o = \tan^{-1}\frac{2l(\omega_o-\omega_i)}{w_f(\omega_o-\omega_i)+w_r(\omega_o+\omega_i)}\,. \tag{3.4}$$

The distance from the center of rotation to the center of the rear wheel gauge described as (R_1) is defined by equation 3.5:

$$R_1 = \frac{w_r(\omega_o+\omega_i)}{2(\omega_o-\omega_i)}\,, \tag{3.5}$$

substituting the value (R_1) into equations 3.3 and 3.4 results in the values of the tangent of the inter and extern steering angles, according to equations 3.6 and 3.7 respectively. The tangent of the internal angle is given by equation 3.6:

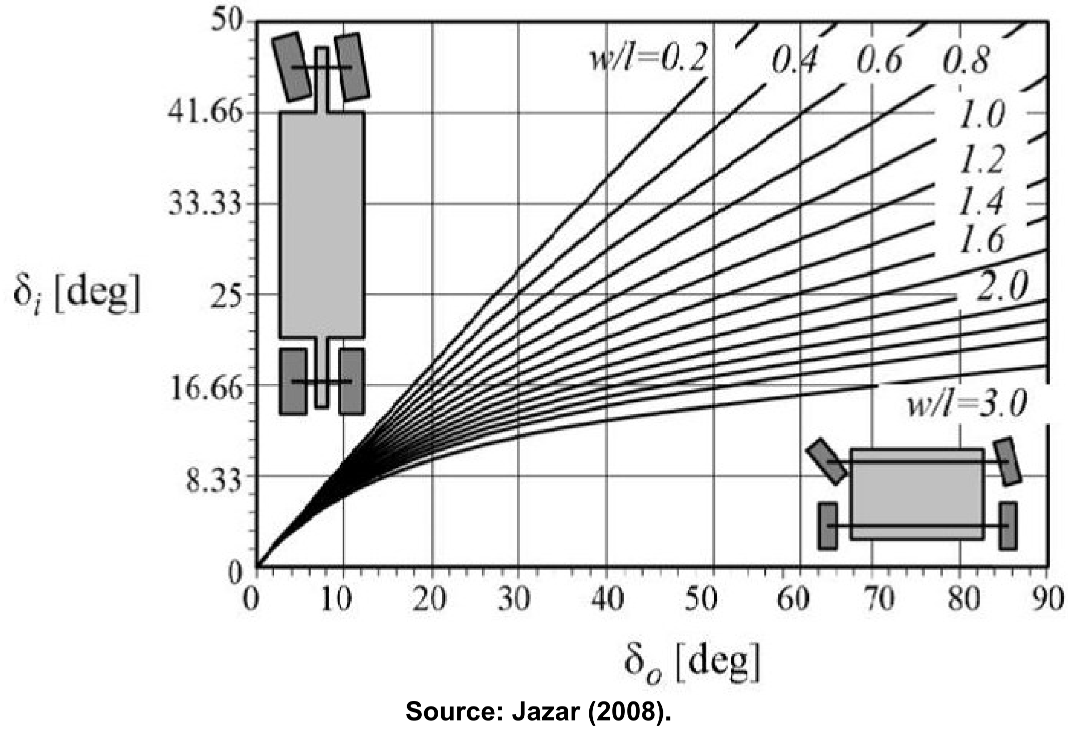
$$\tan \delta_i = \frac{l}{R_1 - \frac{w_f}{2}} , \tag{3.6}$$

end the tangent of the external angle is given by equation 3.7:

$$\tan \delta_o = \frac{l}{R_1 + \frac{w_f}{2}} . \tag{3.7}$$

If the front gauge or distance between wheels on the same axle (w_f) and the rear gauge or distance between wheels on the same axle (w_r) have the same dimensions, a graphical comparison is considered to analyze the value of the intern and extern steering angle, based on the distance between wheels on the same axle (w_f) divided by the Length between the axis (l) based on Ackerman's geometry according to Figure 9.

Figure 9 - Relationship (w/l) based on Ackerman geometry.



Source: Jazar (2008).

The lower the ratio (w/l) the smaller the difference between the internal or external steering angles, the reverse occurs for higher values.

However, each automobile industry adopts an operating range for the relation (w/l) taking into consideration the type of car.

3.3 Kinematic modeling of the 4WS

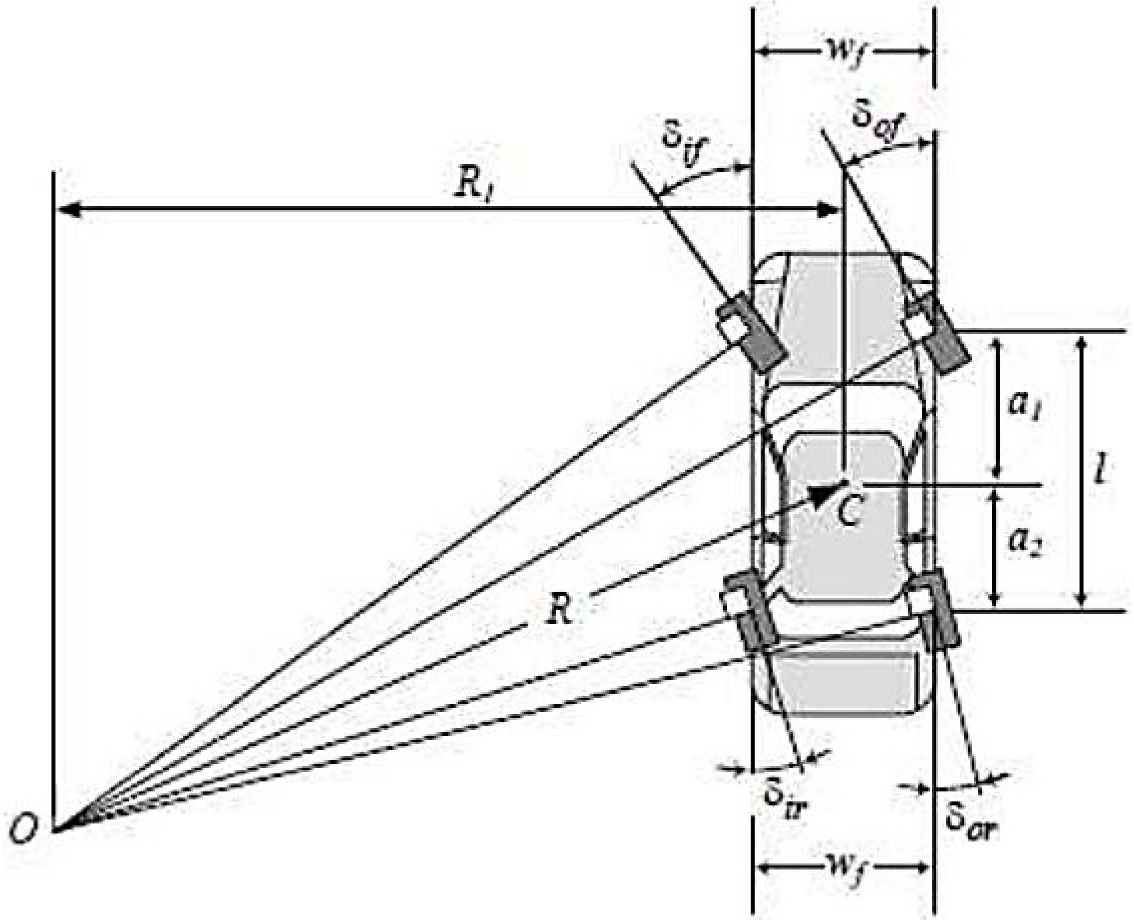
Due to the great complexity of maneuvering, the racing vehicles called 4-Wheel Steering (4WS) have 4-wheel traction and therefore the economic cost is higher compared to FWS vehicles.

For the kinematic analysis of a 4WS vehicle, it follows the same context applied to FWS vehicles with some peculiarities and additional parameters by considering that the rear wheels are subject to a rotation movement along the vertical axis, all this change defines geometric positions such as; the center of mass rotation and the angular acceleration of the vehicle.

The 4WS vehicle models can be classified into positive and negative according to the direction of the wheels.

The Figure 10 illustrates the positive wheel steering 4WS model.

Figure 10 - Positive wheel steering 4WS model.



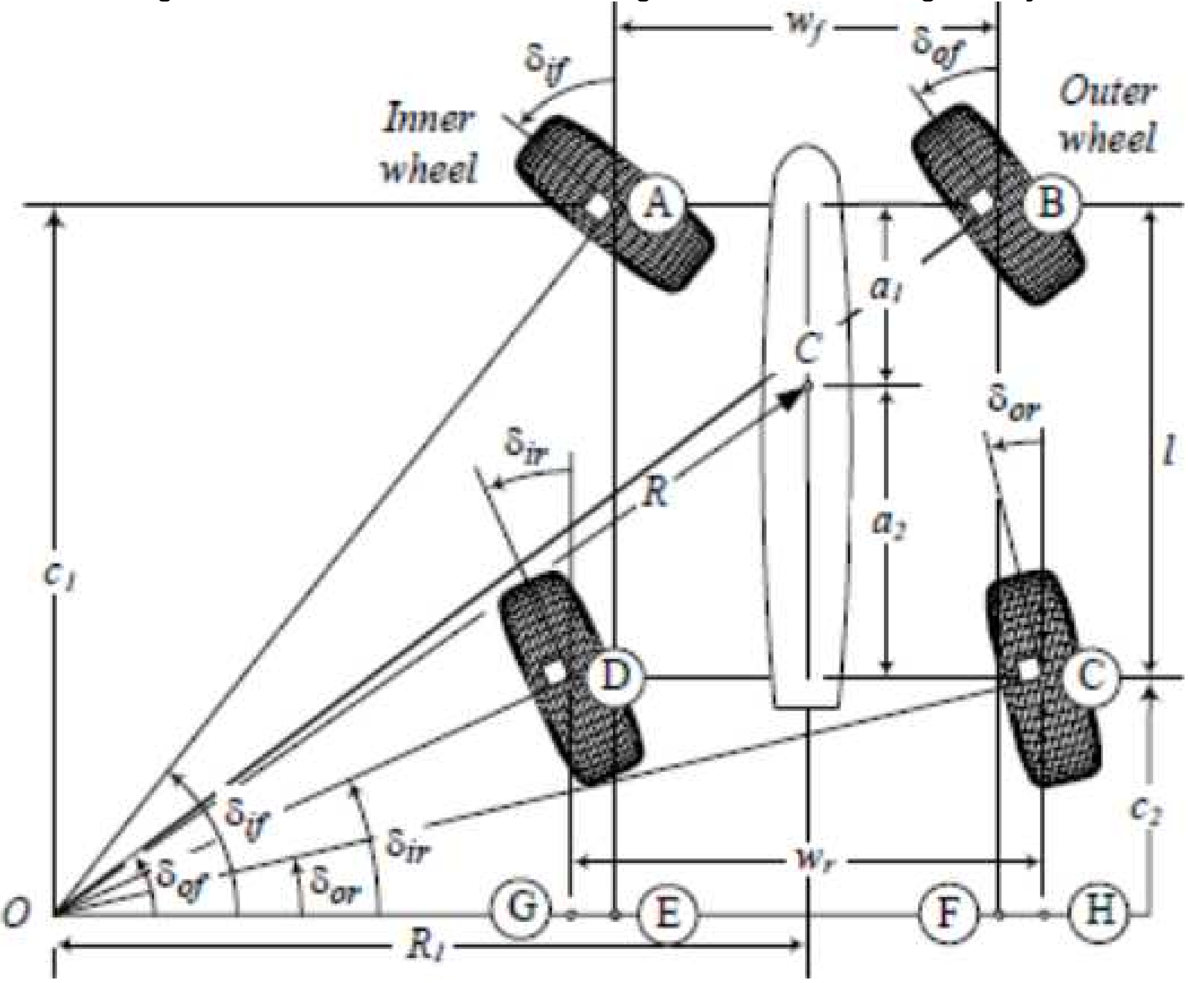
Source: Jazar (2008).

In geometric form, the positive model (4WS) is characterized by equivalence of steering angles as described in equation 3.8:

$$\cot\delta_{of} - \cot\,\delta_{if} = \frac{w_f}{l} - \frac{w_r}{l}\frac{\cot\delta_{of}-\cot\delta_{if}}{\cot\delta_{or}-\cot\delta_{ir}}. \tag{3.8}$$

The Figure 11 shows the vehicle model (4WS) with positive steering when based on the Ackerman geometry.

Figure 11 - Positive vehicle 4WS steering based on Ackerman geometry.



Source: Jazar (2008).

The positive steering model (4WS) based on the Ackerman geometry; the front internal steering angle is determined by equation 3.9:

$$\tan \delta_{if} = \frac{c_1}{R_1 - \frac{w_f}{2}}, \tag{3.9}$$

and the front external steering angle is defined by equation 3.10:

$$\tan \delta_{of} = \frac{c_1}{R_1 + \frac{w_f}{2}}, \tag{3.10}$$

therefore, the geometric equations of the rear wheels are highlighted in equations 3.11 and 3.12.

Rear internal steering angle is represented by equation 3.11:

$$\tan\delta_{ir} = \frac{c_2}{R_1 - \frac{w_r}{2}}\ , \tag{3.11}$$

and for rear external steering angle, is determined the equation 3.12:

$$\tan\delta_{or} = \frac{c_2}{R_1 + \frac{w_r}{2}}\ , \tag{3.12}$$

the cotangent is the inverse of the tangent, which is the ratio between the opposite side of a given acute angle of a rectangle triangle and the adjacent side of the same angle.

However, the difference between the internal and external angles of the front wheels is based on the calculation of the cotangent, as described in equation 3.13:

$$\cot\delta_{of} - \cot\delta_{if} = \frac{w_f}{c_1}\ , \tag{3.13}$$

for internal and external angles of the rear wheels is based on the calculation of the cotangent, as described in equation 3.14:

$$\cot\delta_{or} - \cot\delta_{ir} = \frac{w_r}{c_2}\ . \tag{3.14}$$

The value of R_1 in relation to the front wheels is calculated from the equality of equations 3.9 and 3.10 respectively, as described in equation 3.15:

$$R_1 = \frac{1}{2}w_f + \frac{c_1}{\tan\delta_{if}} = -\frac{1}{2}w_f + \frac{c_1}{\tan\delta_{of}}\ , \tag{3.15}$$

the value of R_1 in relation to the rear wheels is calculated from the equality of equations 3.11 and 3.12 respectively, as described in equation 3.16:

$$R_1 = \frac{1}{2} w_r + \frac{c_2}{\tan \delta_{ir}} = -\frac{1}{2} w_r + \frac{c_2}{\tan \delta_{or}} \, , \tag{3.16}$$

the distance $(c_1,\ c_2,\ l)$ between axes considered from the center of rotation (o) is characterized by equation 3.17:

$$(c_1 = l + c_2) \, , \tag{3.17}$$

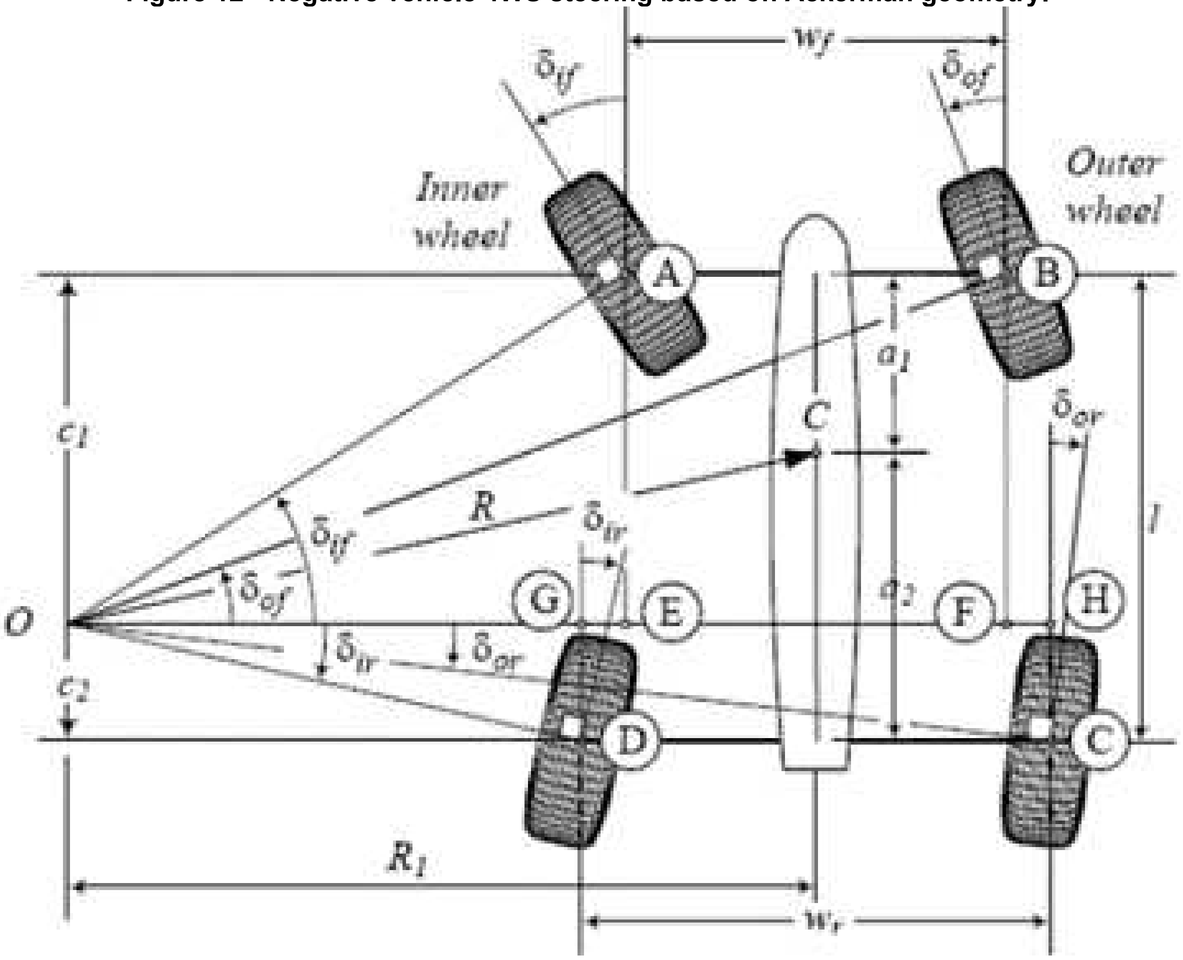
Isolating the distance (c_1) and (c_2) from the equations 3.13 and 3.14 respectively, the distance (l) is defined by equation 3.18:

$$\frac{w_f}{\cot_{of} - \cot_{if}} - \frac{w_r}{\cot_{or} - \cot_{ir}} = l \, , \tag{3.18}$$

the equations presented above, are applied for 4WS vehicles rated positive and negative according to wheel steering.

The 4WS vehicle classified negative according to wheel steering is shown in Figure 12.

Figure 12 - Negative vehicle 4WS steering based on Ackerman geometry.



Source: Jazar (2008).

The position of 4WS vehicles is considered positive when the 4-wheels have the same steering (Figure 11), if the 4-wheels do not have the same steering it is considered negative (Figure 12).

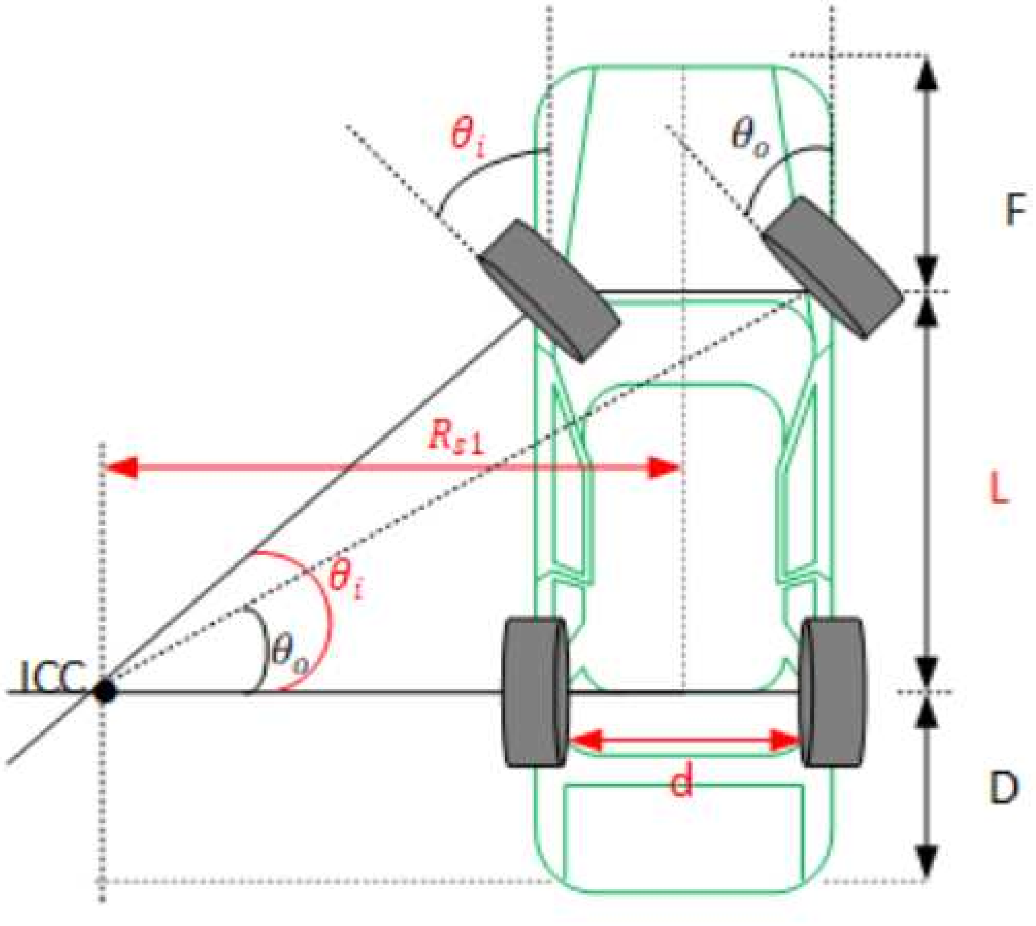
Is describes the geometry of the automatic parking in the next chapter (4).

4 GEOMETRY FOR THE AUTOMATIC PARKING CONTROL

This chapter (4) is presented the geometry of the automatic parking based on the Ackerman steering calculation; where it determines a reference point (ICC) also called the instantaneous center of curvature from the 4-wheels of the car.

For the Ackerman steering calculation, the traction is determined only on the front 2-wheels and on the rear 2-wheels a slip angle is considered which depends on the movement and direction of the front 2-wheels, as shown in Figure 13.

Figure 13 - Geometry for the calculation of the Ackerman direction.



Source: Manuel *et al* (2020).

However, Figure 13 illustrates Ackerman steering from the geometry to the variables that model the initial and final control during the parking maneuver in parallel and perpendicular form.

The variables presented at geometry for the calculation of the Ackerman direction (Figure 13) are described in Frame 3 for further clarification.

Frame 3 - Variables used to calculate the Ackerman direction.

Variables	Descriptions
$\boldsymbol{\theta_i}$	Vehicle's internal steering angle
$\boldsymbol{\theta_o}$	Vehicle's external steering angle
F	Front axle space to vehicle start
R_{s1}	First radius of curvature of the steering angle
L	Distance from front axle to rear axle
ICC	Instant curvature center
d	Internal width between axles (rear)
D	Rear axle space to the end of the vehicle

Source: Own author (2023).

4.1 Initial and final radius during the maneuver

The steering angle is based on the initial and final radius during the maneuver, with the front 2-wheels being responsible for determining the car's position.

The first steering angle is characterized as negative in relation the position (as illustrated in Figures 12 and 13) because the front 2-wheels do not have the same direction as the rear 2-wheels.

To calculate the (R_{s1}) first radius of curvature of the steering angle, the equation 4.1 is defined as the solution, where (tan) is named as tangent;

$$R_{s1} = \frac{L}{\tan(\theta_{i1})} + \frac{d}{2}, \tag{4.1}$$

and to calculate the (R_{s2}) second/final radius of curvature of the steering angle, equation 4.2 is defined as the solution, where (tan) is named as tangent;

$$R_{s2} = \frac{L}{\tan(\theta_{i2})} + \frac{d}{2}, \tag{4.2}$$

the value of (R_{s1}) and (R_{s2}) varies according to the dimensions of the vehicle from a fixed value of vehicle steering/steering angle (θ_{i1}) and (θ_{i2}) respectively. Where, the distance from the front axle to the rear axle (L) and the distance between the rear axle (d) are constant.

An important relationship between the internal and external steering angle is represented by equation 4.3;

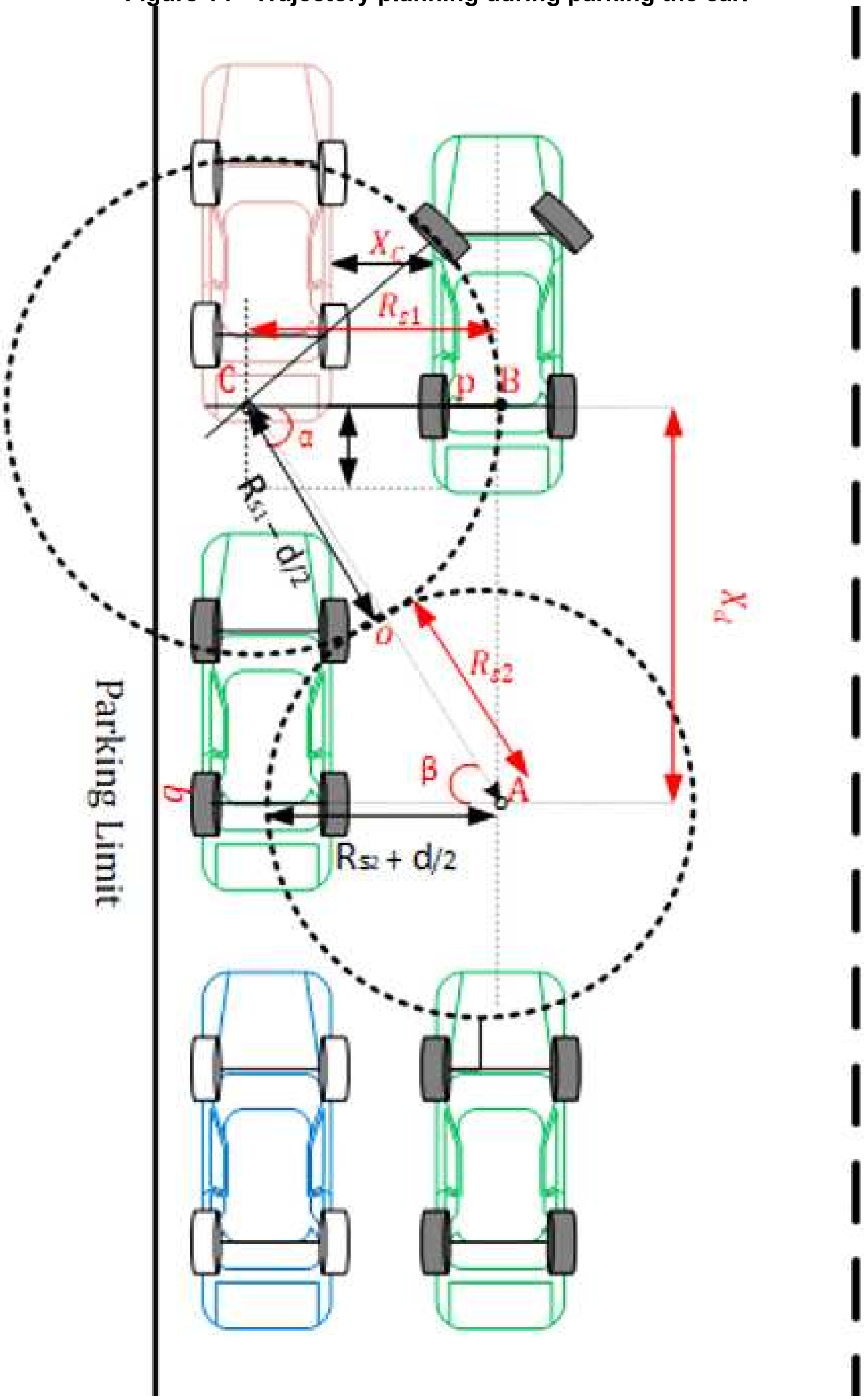
$$\cot\theta_i - \cot\theta_o = \frac{d}{L}, \tag{4.3}$$

the subtraction or difference respectively of the cotangents (Cot) of the intern and extern steering angles of the car, is equal to the distance between the rear axle (d) divided by the distance from the front axle to the rear axle (L).

4.2 Trajectory planning for parking the car

The development of the maneuver assist algorithms during parallel and perpendicular parking is determined from the trajectory planning of the car according to Figure 14.

Figure 14 - Trajectory planning during parking the car.



Source: Manuel *et al* (2020).

Where (X_d) is the size of the parking vacancy and (X_c) is the distance between the main car (green) and the parked car (red); call too as safety factor so that the car doesn't collide during the initial maneuver radius.

After the execution of the first curvature radius (R_{s1}), the first position of the main car (green) is designated as $(R_{s1} - d/2)$; The direction of rotation of the first curvature radius was adopted and considered clockwise (negative) under the influence of the initial position of the main car (green), which is $(-d/2)$.

However, after the execution of the second radius of curvature (R_{s2}), the second/final position of the main car (green) is designated as $(R_{s2} + d/2)$, where (d) is the intern width between the axles-rear; The direction of rotation of the second radius was adopted and considered counter-clockwise (positive) under the influence of the final position of the main car (green), which is $(+d/2)$.

From the second radius it is possible to determine the distance of the points (Aq), according to equation 4.4:

$$Aq = R_{S2} + d/2\ . \tag{4.4}$$

4.3 Pythagorean theorem

The rectangle triangle ABD shown in Figure 14 is part of the trigonometric solution, where the square root of the opposite side added by the square root of the adjacent side is equivalent to the square root of the hypotenuse (ZHELTIKOV, 2022).

The representation of the Pythagorean theorem is based on equation 4.5:

$$CB^2 + AB^2 = AC^2\ , \tag{4.5}$$

by trigonometric equivalence the distance between the points of the right triangle ABC is determined by equation 4.6:

$$X_d{}^2 = 2R_{s1}{}^2 - R_{s1}{}^2\ , \tag{4.6}$$

replacing equation 4.6 in the equation 4.5, it is possible to calculate the value of (R_{s2}) as described in equation 4.7:

$$R_{s1}{}^{2} + X_{d}{}^{2} - R_{s1}{}^{2} = R_{s2}{}^{2} \ , \qquad (4.7)$$

and the angle (α) is obtained from equation 4.8:

$$\alpha = \operatorname{atan}\left(\frac{X_d}{R_{s1}}\right), \qquad (4.8)$$

where $(atan)$ is named arctangent.

The angle (α) is projected between the AC points, the angle (β) is projected onto the opposite rectangle triangle CqA, precisely at the CA points. However, angle (α) is equivalent to angle (β).

The first arc-circle originates from the points (p) and (o) of the center C, its length is represented by equation 4.9:

$$length(\mathrm{po}) = \left(\frac{\alpha}{360}\right) 2\pi \left(R_{s1} - \frac{d}{2}\right), \qquad (4.9)$$

the second/final arc-circle originates from the points (o) and (q) of the center A, its length is represented by equation 4.10.

$$length(oq) = \left(\frac{\beta}{360}\right) 2\pi \left(R_{s2} + \frac{d}{2}\right). \qquad (4.10)$$

When substituting of equation 4.7 by equation 4.2, makes it possible to calculate the value of (θ_{i2}) according to equation 4.11:

$$\theta_{i2} = \tan^{-1}\left(\frac{L}{R_{s2} - d/2}\right), \qquad (4.11)$$

initially the value of the steering angle (θ_{i1}) is determined, so the sum of the two intern angles of the car's steering referring to (R_{s1}) and (R_{s2}) is characterized by equation 4.12:

$$\theta_i = \theta_{i1} + \theta_{i2} \,, \tag{4.12}$$

when substituting of equation 4.12 by equation 4.3, makes it possible the value of the external steering/steering angle (θ_o) according to equation 4.13:

$$\theta_o = \mathrm{Cot}^{-1}(\mathrm{Cot}\theta_i - d/L) \,, \tag{4.13}$$

as proof of the calculations mentioned here, the calculation of cotangent is a way to certify that the vehicle will align its wheel obeying the new steering angle configurated and will successfully finish the final trajectory.

In the next chapter (5) the main parameters used to develop the parking algorithms is described.

5 VALIDATION OF THE MANEUVER ASSIST ALGORITHMS

In this chapter (5) is described the main parameters used to develop the parking algorithms based on the equations presented previously.

Some global car parameters must be considered to obtain a successful response during the parking maneuver in the parallel and perpendicular form, highlighted: the change of the steering angle, position of the main car, parked cars, first radius of curvature, second radius of curvature and the distance to park the car.

The global parameters of the main car (green), described in Table 1, were used:

Table 1 - Global car parameters.

Variables	Descriptions	Measures	Units
$\boldsymbol{\theta_{i1}}$	Vehicle's internal steering angle	$45° = 0{,}7854\,rad$	rad
$\boldsymbol{\theta_o}$	Vehicle's external steering angle	----	----
$\mathrm{d_o}$	Total vehicle width	15	$dimensionless$
B	Total length of the vehicle	33	$dimensionless$
d	Internal width between axles (rear)	5	$dimensionless$
D	Rear axle space to the end of the vehicle	5	$dimensionless$
F	Front axle space to vehicle start	8	$dimensionless$
L	Distance from front axle to rear axle	20	$dimensionless$
R_{s1}	First radius of curvature of the steering angle	----	----
ICC	Instant curvature center	----	----

Source: Own author (2023).

Obs: The global parameters shown in Table 1 may vary depending on the car model. Therefore, the parameters are originally defined by the automotive industries, and can be adapted according to the type of project.

The radian measure is dimensionless, that is, it does not depend on the unit of measure with which arc lengths are measured, therefore it is indifferent to the measure with which these lengths are measured (in, mm, cm, m, among other units).

Since this is a computational simulation, the unit of the quantities is indifferent, remembering that if the unit (m) is chosen it is impossible on a computer display to view the drawing as a whole; In this doctoral thesis chose to name the units as dimensionless according to Table 1.

For validation of the parking algorithm in parallel form, it was that the value of (θ_{i1}) is equal to $45°$ or $0{,}7854\ rad$ since in trigonometry a circle travels 360°, equivalent to 2π (rad). The unit (rad) refers to radians and the unit $(°)$ refers to degrees.

The value of (R_{s1}) is obtained from equation 4.1 by substituting the values of the Table 1, the result is equation 5.1:

$$R_{s1} = 22.5\,, \tag{5.1}$$

and the value of (X_d) is obtained from equation 4.6 by substituting the values of the equation 5.1, the result is equation 5.2:

$$X_d \cong 39\ . \tag{5.2}$$

Substituting the values of equations 5.1 and 5.2 by equation 4.7, the value of (R_{s2}) is described in equation 5.3:

$$R_{s2} = 39\,. \tag{5.3}$$

Substituting the values of equations 5.1 and 5.2 by equation 4.8, the value of angle (α) is described in equation 5.4:

$$\alpha = 1.047\,rad = \beta\ . \tag{5.4}$$

When replaced the equations 5.1 and 5.4 by equation 4.9, is determined the value of the length of the first arc-circle, represented to equation 5.5:

$$length(\mathrm{po}) = 0.365\,, \tag{5.5}$$

when replaced the equations 5.3 and 5.4 by equation 4.10, is determined the value of the length of the second arc-circle, represented to equation 5.6:

$$length(oq) = 0.758\,. \tag{5.6}$$

Substituting the value of equation 5.3 by equation 4.2 the value of the second steering angle (θ_{i2}) is determined in the equation 5.7:

$$\theta_{i2} = 0.5012\,rad\,. \tag{5.7}$$

From the value of the steering angle (θ_{i1}) of the first radius determined in Table 5.1, it is possible to calculate the car's internal steering angle (θ_i). When replaced the values of equation 5.7 and values of the steering angle (θ_{i1}) by equation 4.12, the value of the steering angle (θ_i) is represented by equation 5.8:

$$\theta_i = 1.2866\,rad\,, \tag{5.8}$$

and the value of the external steering angle (θ_o) is obtained when is substituting the value of equation 5.8 by equation 4.3; however, the value of the external steering angle (θ_o) is described in the equation 5.9:

$$\theta_o = 1.528\,rad\,. \tag{5.9}$$

When is substituted the values of equations 5.8 and 5.9 by equation 4.3 the result is presented in the equation 5.10:

$$0.25 = 0.25 \tag{5.10}$$

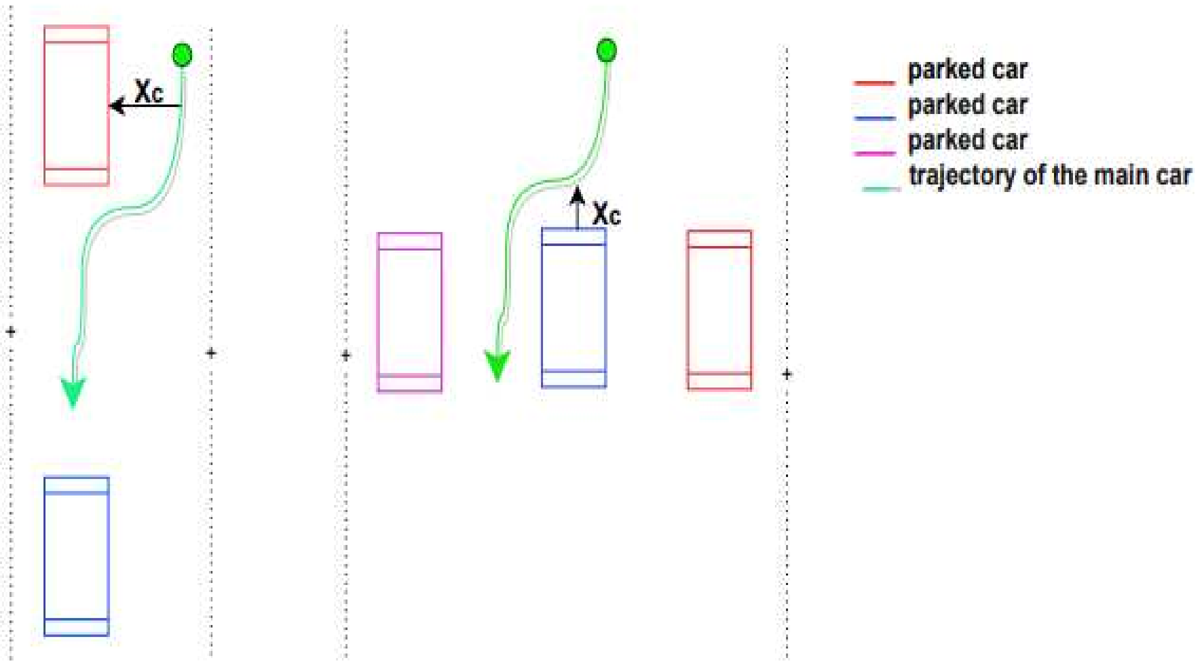
As proof of the calculations mentioned here, the calculation of cotangent is a way to certify that the vehicle will align its wheel obeying the new steering angle configurated and will successfully finish the car's final trajectory.

As a trajectory planning strategy, was first developed the parallel parking algorithm, then modified the position of the cars (to a perpendicular form) with the same geometric concept of the initial and final maneuver radius, but with some numerical adjustments to achieve maximum precision and accuracy during the parking maneuver.

However, a generic workflow of the main car's trajectory was initially adopted, as per Figure 15,

Figure 15 - Generic workflow of the main car trajectory: (a) trajectory in parallel form, (b) trajectory in perpendicular form, (c) legends.

(a) (b) (c)



Source: Own author (2023).

from this generic workflow (Figure 15) it is possible to measure or classify errors classified as disturbance during the (parallel) trajectory of the main car relative to the parked car.

The error is determined when the distance (Xc) is smaller or larger according to equation 5.11:

$$3 \leq \ \mathrm{Xc} \ \geq 3\,, \qquad (5.11)$$

where; (Xc) is the distance between the main car and the parked car and 3 is the (dimensionless) value assigned to the distance (Xc) in the simulation.

The parameters used to classify the errors/disturbances during the trajectory of the main car and the parked car, was determined in Table 2.

Table 2 - Parameters used to classify the errors/disturbances during the trajectory of the car.

Parked car	Main car	Time (s)	Units
0	33	0	dimensionless
0	61	5	dimensionless
15	18	10	dimensionless
15	61	20	dimensionless
70	18	50	dimensionless
100	94	60	dimensionless
100	33	80	dimensionless
70	94	100	dimensionless

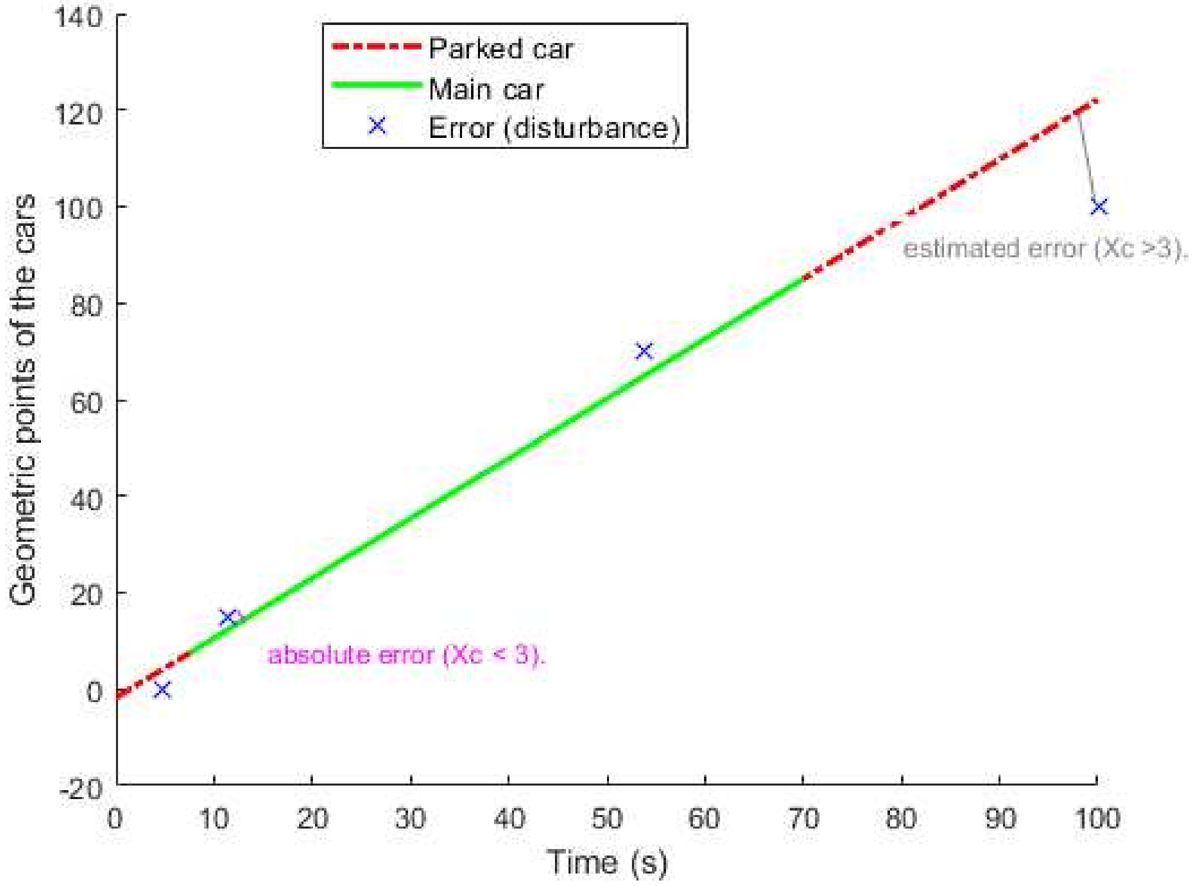
Source: Own author (2023).

The absolute error is described when (Xc) is less than 3, as shown in equation 5.12:

$$\mathrm{Xc} < 3\,. \tag{5.12}$$

The absolute error is irreversible, since this condition indicates that the car collided in lateral form with the parked car, as shown in Graphic 3,

Graphic 3 - Classification of the absolute and estimated error during the trajectory of the main car.



Source: Own author (2023).

the estimated error is described when (Xc) is greater than 3, as shown in equation 5.13:

$$\mathrm{Xc} > 3\ . \tag{5.13}$$

In this case it is a reversible error, since this condition indicates that the main car is far away from the parked car and the space it intends to park in.

The equation 5.14:

$$\mathrm{Xc} = 3\ , \tag{5.14}$$

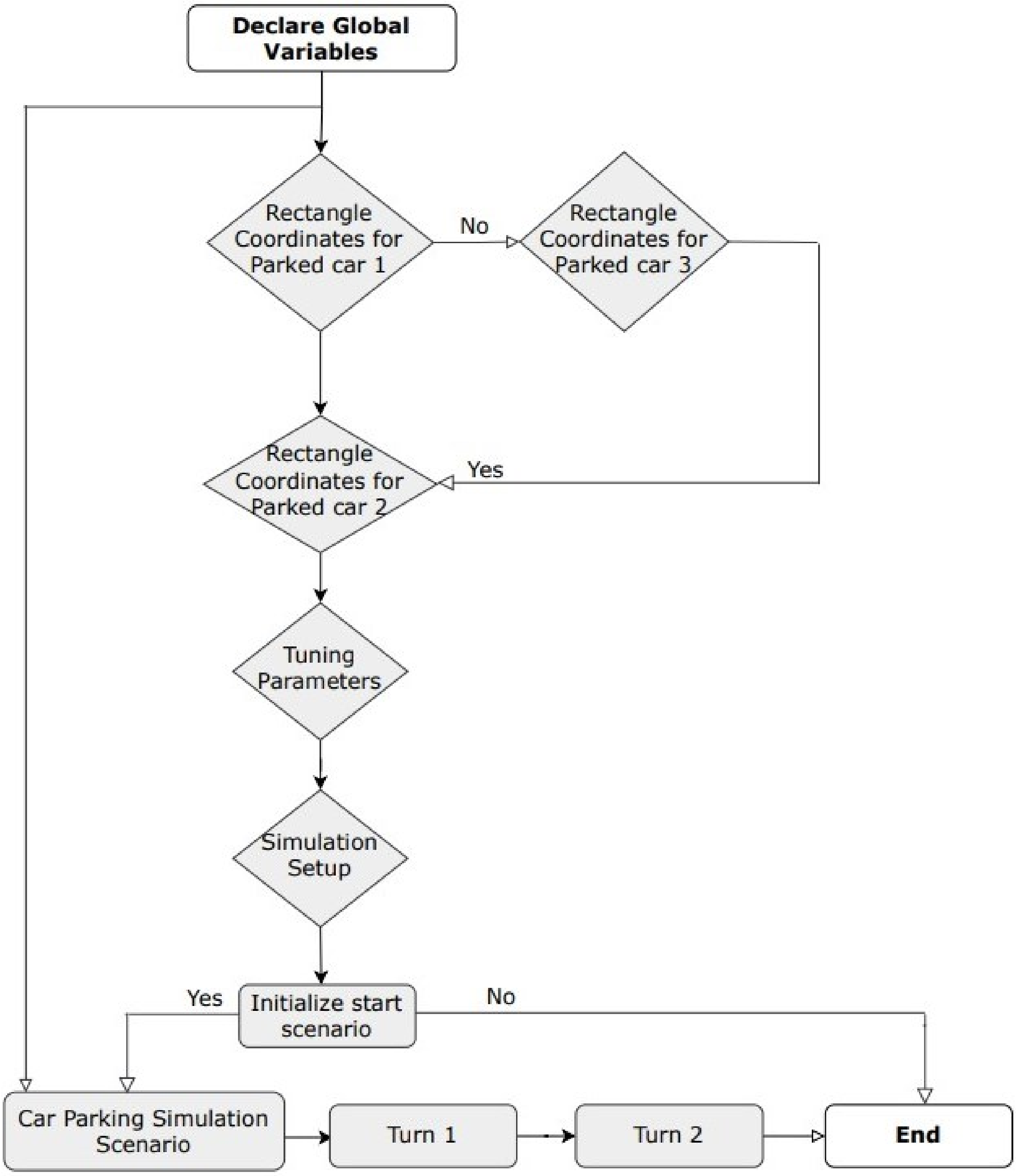
ensured that there was no collision during the parking maneuver between the main car and the parked car.

Similarly, the strategy of measuring or classifying errors/disturbances was adopted for car trajectory in the perpendicular format.

5.1 Results and discussions of the simulations

The results presented in this subchapter (5.1) are based on the automatic parking algorithms, highlighted as Parallel and Perpendicular; which includes a workflow for the best execution guidelines, from global variables to calling functions for the correct trajectory of the car, as shown in Figure 16.

Figure 16 - Workflow used in the development of the algorithms.



Source: Own author (2023).

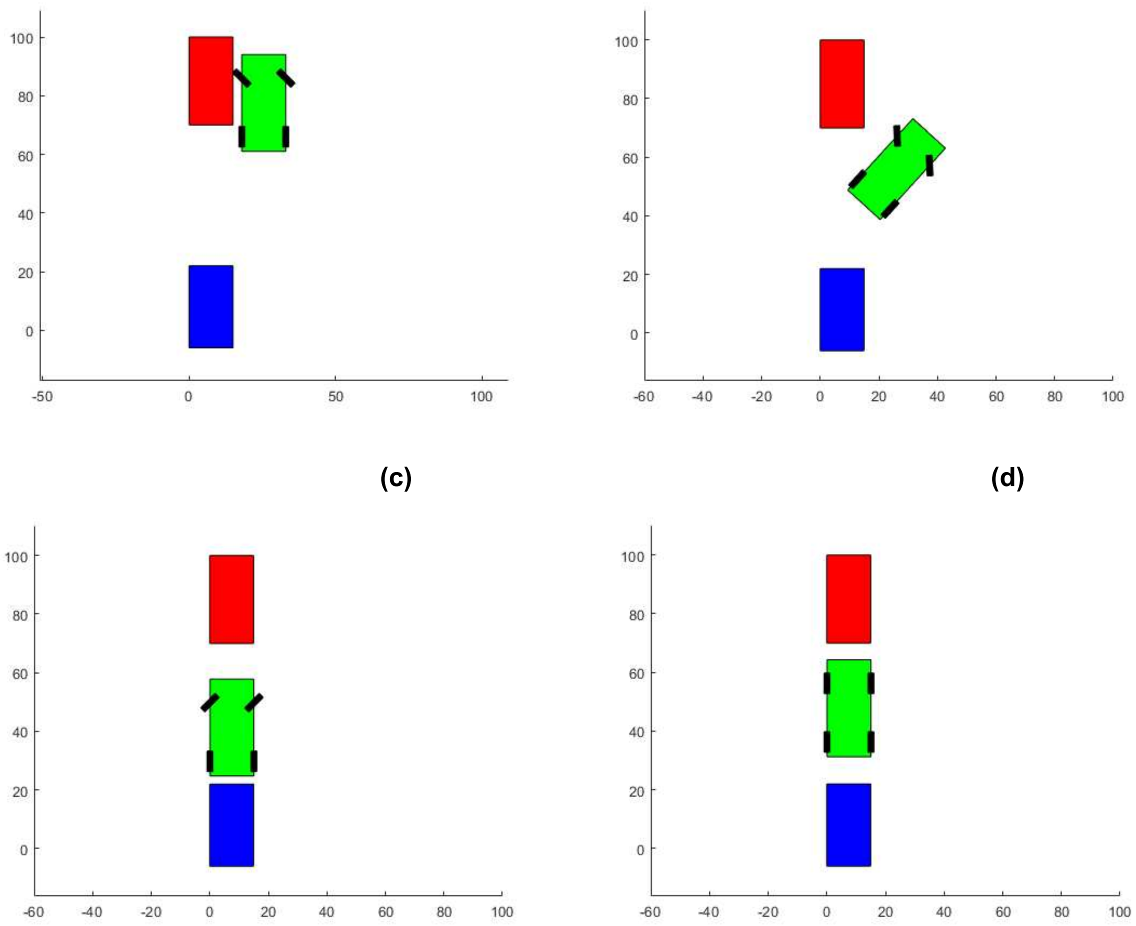
The results of the algorithms for maneuvering the car in parallel and perpendicular form are shown in the sequence.

- **Parallel Parking**

The simulation results that characterize parallel parking of the car are shown in Figure 17.

Figure 17 - Parallel parking simulation results: (a) 1st position of the main car (green), (b) 2st position of the main car (green), (c) 3st position of the main car (green), (d) 4st position of the main car (green).

(a) **(b)**



Source: Own author (2023).

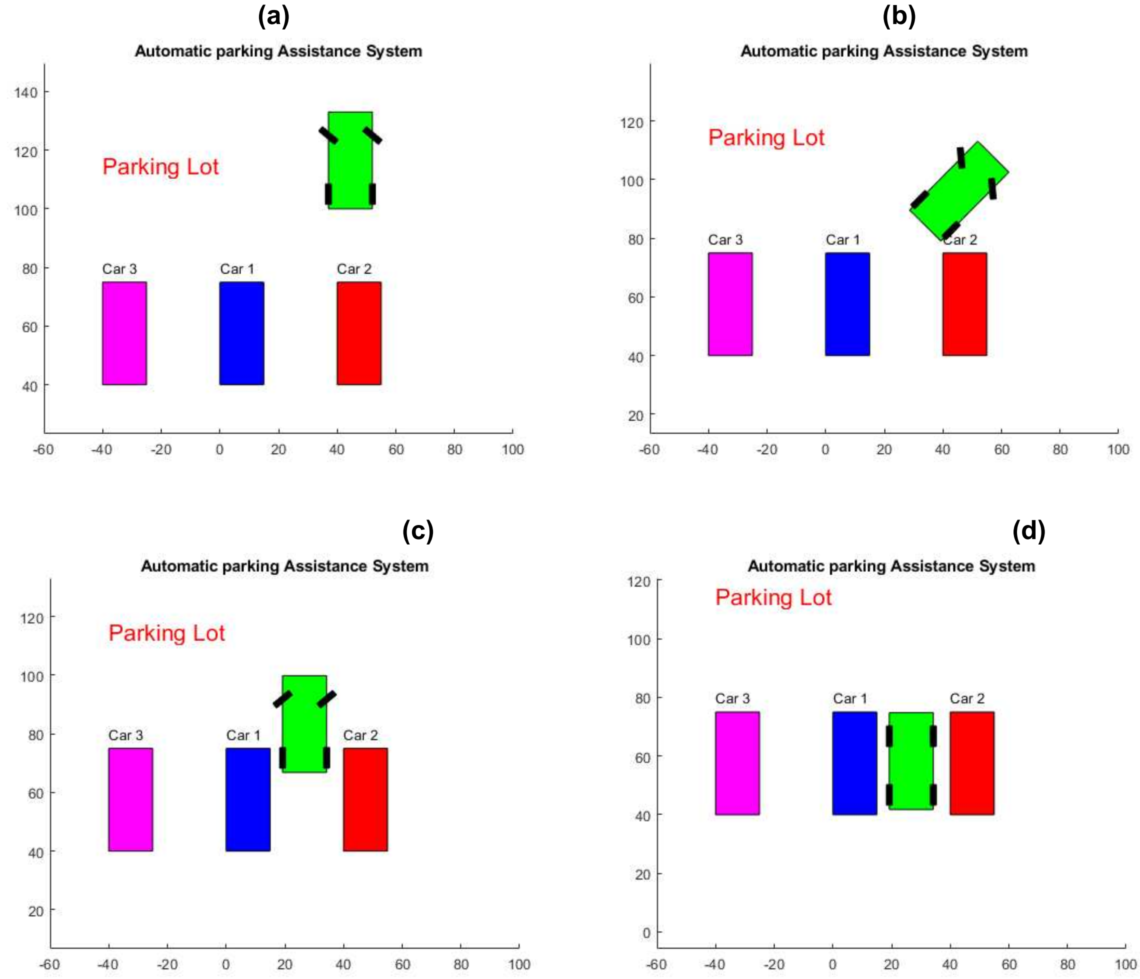
In the 1st position the initial maneuver angle (*45º = 0.0137 rad*) relative to the front wheels is considered; in the 2st position the first circular arc is executed.

However, the 3st position the second arc-circle is executed and the car is successfully parked in the desired vacancy and in the 4st position the front wheels of the car are aligned at (*90°*) in relation the wheelbase based on the geometric equation 3.13.

- **Perpendicular Parking (between cars 1 and 2)**

The simulation results that characterize perpendicular parking of the car are shown in Figure 18.

Figure 18 - Perpendicular parking simulation results: (a) 1st position of the main car (green), (b) 2st position of the main car (green), (c) 3st position of the main car (green), (d) 4st position of the main car (green).



Source: Own author (2023).

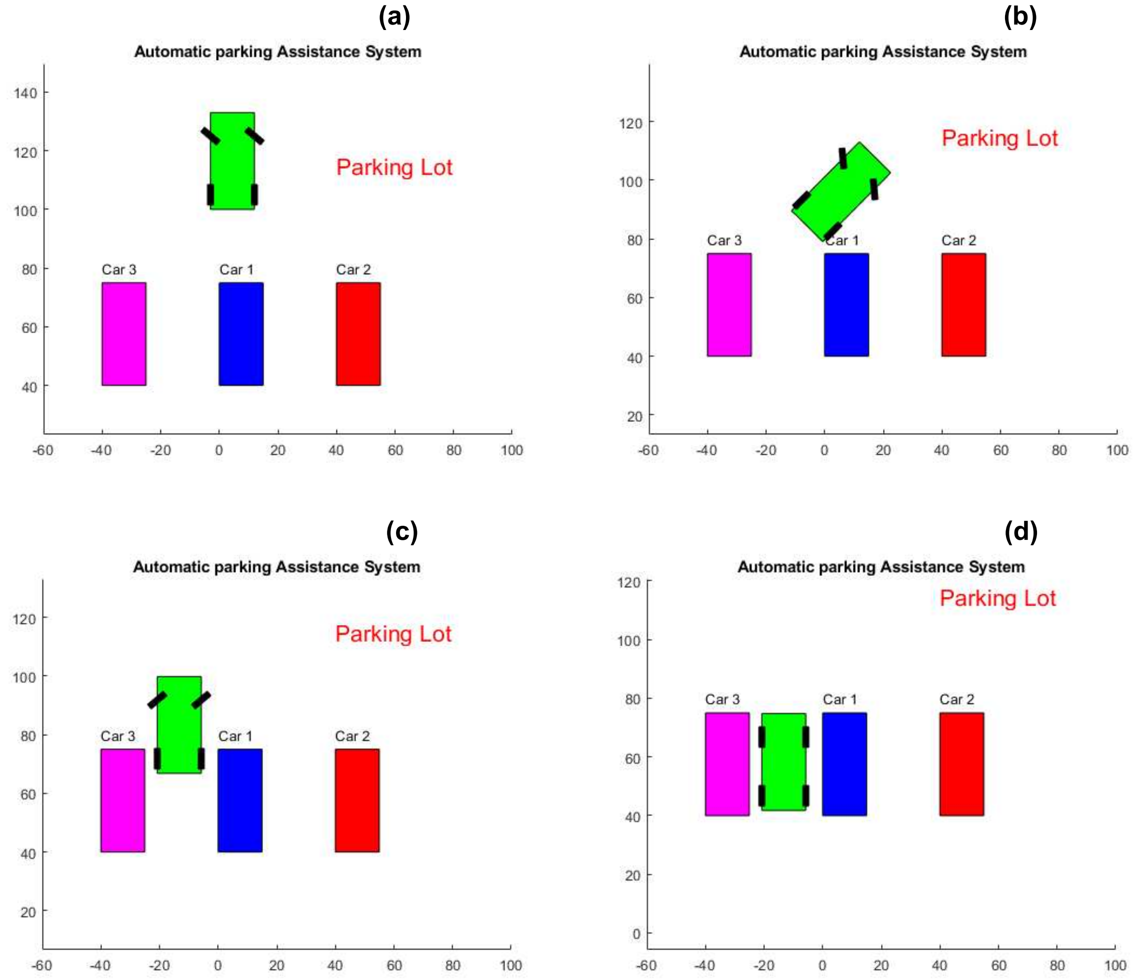
In the 1st position initial maneuver angle (*45º = 0.0137 rad*) relative to the front wheels is considered; in the 2st position the first circular arc is executed.

However, the 3st position the second arc-circle is executed and the car is successfully parked in the desired vacancy and in the 4st position the front wheels of the car are aligned at (*90°*) in relation the wheelbase based on the geometric equation 3.13.

- **Perpendicular Parking (between cars 1 and 3)**

The simulation results that characterize perpendicular parking of the car are shown in Figure 19.

Figure 19 - Perpendicular parking simulation results: (a) 1st position of the main car (green), (b) 2st position of the main car (green), (c) 3st position of the main car (green), (d) 4st position of the main car (green).


(a) **(b)** **(c)** **(d)**

Source: Own author (2023).

In the 1st position initial maneuver angle (*45º = 0.0137 rad*) relative to the front wheels is considered; in the 2st position the first circular arc is executed.

However, the 3st position the second arc-circle is executed and the car is successfully parked in the desired vacancy and in the 4st position the front wheels of the car are aligned at (*90°*) in relation the wheelbase based on the geometric equation 3.13. The equation 4.3 allows full alignment of the car once the steering angle at the end of the maneuver aligns the front 2-wheels with an angle equal to (0º = 0 rad).

All results presented in this chapter, endorse the developed algorithm and how the geometric calculation is fundamental to correctly dimension the initial maneuver radius and the final position of the car.

One form to assign/determine the parallel parking vacancy $(ParP)$ is to consider the parking vacancy equal to the total length of the car plus the safety factor (SF) to guarantee a space in front and rear of the main car in relation to the parked cars), According to equation 5.1.

$$Parp = \left((\mathrm{F} + \mathrm{L} + \mathrm{D}) + (SF)\right). \tag{5.15}$$

One form to assign/determine the perpendicular parking vacancy $(PerP)$ is to consider the parking vacancy equal to the total width of the car plus the safety factor (SF) to guarantee a space in left and right of the main car in relation to the parked cars), According to equation 5.2.

$$Perp = \left((\mathrm{d}) + (SF)\right). \tag{5.16}$$

Obs: The geometric equations allowed the developed algorithm to have precision and accuracy when parking the car parallel or perpendicularly way.

The next chapter (6) describes the final considerations of this doctoral thesis.

6 FINAL CONSIDERATIONS

This chapter (6) describes the final considerations of this thesis, where the geometrical equations presented give rise to the kinematic that directly influence in the dynamics of the car and enables to dimension the initial and final maneuver radius during automatic parallel and perpendicular parking.

The complexity existing in development these maneuver assistance algorithms, includes modeling system dynamics linear and not linear that can be validated in many different application projects; from systems electrical, mechanical, electronic, mechatronic, modeling mathematics, computational and industrial production among others.

The type of programming language used to validate of the algorithms to assist the maneuver was based on in language (C) and presented in language (.m); like great benefits these 2 types of programming language have easy implementation and are compatible with most modules, highlighted the automotive CAN network modules (responsible for communication bus that sends information such as status of sensors, actuators and request to request actuation of the body control modules).

The parking maneuvers algorithms in parallel and perpendicular form validated, can be adapted to different automatic car controls, such as oblique and frontal parking, lane changing, obstacle avoidance, autopilot, longitudinal and lateral control; with the possibility of being implemented with different types of sensing.

As a product in the automotive industry the validation of the parking algorithms ensures safety, reduced human effort and comfort during car maneuvering; according to the simulations results presented.

Therefore, in addition to contributing to the development technological and industrial this thesis validates of the maneuver assistance algorithms with low computational cost and aims at later implementation in cars electric, intelligent, autonomous and others.

6.1 Future considerations

Four options are indicated as future work/research:

1. Development of the automatic parking algorithm in oblique form;
2. Use the equations/parameters presented in this thesis to validate a controller (PID or MPC) of longitudinal and lateral action;
3. Validation of the algorithms based on a 4WS vehicle;
4. *Hardware-in-the-Loop* validation (Prototype/real car).

REFERENCES

BARI, D.; *et al.* Design and manufacturing of a system to measure the turning radius of vehicle. **Int J Eng Sci Adv Technol**, Nashik, v. 4, n. 6, pp. 536-540, nov-dec. 2014.

BHARADWAJ, A. **Evolution of the global automobile industry. Environmental regulations and innovation in advanced automobile technologies:** perspectives from Germany, India, China and Brazil. Singapore: Springer, 2018.

BHOI, S. K.; KHILAR, P. M. Vehicular communication: a survey. **IET networks**, Odisha, v. 3, n. 3, pp. 204-217, sep. 2014.

CUMINGS, B. The Origins and development of the northeast asian political economy: industrial sectors, product cycles, and political consequences. **International Organization**, Cambridge, v. 38, n. 1, pp. 1-40, 1984.

CHANG, C. M.; *et al.* Eyes on a car: an interface design for communication between an autonomous car and a pedestrian. *In:* PROCEEDINGS OF THE 9th INTERNATIONAL CONFERENCE ON AUTOMOTIVE USER INTERFACES AND INTERACTIVE VEHICULAR APPLICATIONS. 2017, Oldenburg: AutomotiveUI '17, 2017. pp. 65-73.

CHELBI, N. E.; GINGRAS, D.; SAUVAGEAU, C. Worst-case scenarios identification approach for the evaluation of advanced driver assistance systems in intelligent/autonomous vehicles under multiple conditions. **Journal of Intelligent Transportation Systems**, London, v. 26, n. 3, pp. 284-310, dec. 2020.

CHOPRA, S.; SODHI, M. Reducing the risk of supply chain disruptions. **MIT Sloan management review**, Cambridge, v. 55, n. 3, pp. 72-80, mar. 2014.

CHU, D.; *et al.* Rollover speed prediction on curves for heavy vehicles using mobile smartphone. **Measurement**, Amsterdam, v. 130, pp. 404-411, dec. 2018.

CHRISTOPHER, M. **Logistics and supply chain management.** 4. ed. Boston: Pearson Uk, 2010.

DE OLIVEIRA ANDRADE, K.; HERNANDES, A. C.; BECKER, M. A rule-based controller simulation for an autonomous parallel parking of a car-like robot using laser sensors. *In:* 21th INTERNATIONAL CONGRESS OF MECHANICAL ENGINEERING. 2012, Natal: Proceedings of COBEM, 2012. pp. 1042.

ERRAJI, O. **Model predictive control system design of a passenger car for valet parking scenario.** 2018. Master Thesis - Department of Mechanical, Automotive, and Materials Engineering, University of Windsor, Ontario, 2018.

EXAME. 20 sentences by Albert Einstein to understand life, science and art. **Science.** 29 may. 2021. Available at: https://exame.com/ciencia/20-frases-de-albert-einstein/. Accessed on: 22 Aug. 2022.

FORRESTER, R. The invention of the internal combustion engine and the motor car. 04 feb. 2020. Available at: https://hcommons.org/deposits/item/hc:28591/. Accessed on: 09 Aug. 2022.

FUJIMOTO, T. The long tail of the auto industry life cycle. **Journal of Product** Innovation Management, Tokyo, v. 31, n. 1, pp. 8-16, jan. 2014.

GUPTA, A.; DIVEKAR, R. Autonomous parallel parking methodology for Ackerman configured vehicles. **ACEEE International Journal on Communication**, Washington (D.C), v.1, n. 2, pp. 1-6, jul. 2010.

GUPTA, C.; *et al.* Automatic tractor slip-draft embedded control system. **Computers and Electronics in Agriculture**, Amsterdam, v. 165, pp. 104947, oct. 2019.

GUERRERO-IBÁÑEZ, J.; ZEADALLY, S.; CONTRERAS-CASTILLO, J. Sensor technologies for intelligent transportation systems. **Sensors**, Basel, v. 18, n. 4, pp. 1212, apr. 2018.

HEINEN, M. R.; *et al.* SEVA3D: Autonomous vehicles parking simulator in a three-dimensional environment. **INFOCOMP Journal of Computer Science**, São Leopoldo (RS), v. 6, n. 2, pp. 63-70, oct. 2007.

JAZAR, R. N. **Vehicle dynamics:** Theory and Application. 1. ed. New York: Springer Science and Business Media, 2008.

JAZAR, R. N. **Vehicle dynamics:** Theory and Application. 3. ed. Cham: Springer International Publishing, 2017.

JEANNERET, B.; *et al.* A novel tool to test control strategy with human and hardware in the loop. *In:* VEHICLE POWER AND PROPULSION CONFERENCE (VPPC), 2017, Belfort: IEEE, 2017. pp. 1-5.

KAIWARTYA, O.; *et al.* Internet of vehicles: motivation, layered architecture, network model, challenges, and future aspects. **IEEE Access**, Viena, v. 4, pp. 5356-5373, sep. 2016.

KUPREŠAK, M. **Parking assistance software.** 2020. Doctoral Thesis - Department of Multimedia Systems and Digital Television, University of Osijek, Osijek, 2020.

MANUEL, C. J. T.; SANTOS, M. M. D.; TUSSET, A. M. Mathematical modeling attributed to kinematics and dynamics of a vehicle with 4-wheels. **The European Physical Journal Special Topics**, Switzerland, v. 230, n. 18-20, pp. 3663-3672, jul. 2021.

MANUEL, C. J. T.; *et al.* Geometric aspects to perform maneuver in the automatic parking system. *In:* XI CONGRESSO BRASILEIRO DE ENGENHARIA DE PRODUCÃO: ENGINEERING IN SMART CITIES. 2021, Ponta Grossa: ConBRepro, 2021.

MANUEL, C. J. T.; *et al.* Predictive control applied to the steering system of an autonomous vehicle. **Journal of Vibration Engineering and Technologies**, Switzerland, v. 10, n. 6, pp. 2275-2282, may. 2022.

MANUEL, C. J. T.; *et al.* Computational validation of the best tuning method for a vehicle-integrated PID controller. **Modelling and Simulation in Engineering**, Toronto, v. 2022, jun. 2022.

MANUEL, C. J. T.; *et al.* Longitudinal modeling of a road vehicle: 4-wheel traction. **International Journal of Robotics and Control Systems**, Yogyakarta, v. 2, n. 2, pp. 357-369, jun. 2022.

MANUEL, C. J. T.; *et al.* **Design and development of algorithms and control strategies for automatic parking systems:** advanced driver assistance system. 2020. Master Thesis - Graduate Program in Electrical Engineering, Universidade Tecnológica Federal do Paraná, Ponta Grossa, 2020.

MARTÍNEZ-DÍAZ, M.; SORIGUERA, F.; PÉREZ, I. Autonomous driving: a bird's eye view. **IET Intelligent Transport Systems**, London, v. 13, n. 4, pp. 563-579, dec. 2019.

METIN, F.; SEZER, V. Analysis and improvement of geometric parallel parking methods with respect to the minimum final lateral distance to the parking spot. **International Journal of Intelligent Transportation Systems Research**, Switzerland, v. 19, n. 2, pp. 441-455, apr. 2021.

MINISTRY OF INFRASTRUCTURE. National registry of traffic accidents and statistics. **Accidents.** 13 apr. 2023. Available at: https://www.gov.br/infraestrutura/pt-br/assuntos/transito/arquivos-senatran/docs/renaest. Accessed on: 08 May. 2023.

MOON, H. K.; *et al.* A study on the concept design of 4-wheel independent steering system and driving mode. **SAE Technical Paper**, United states, pp. 8, mar. 2022.

MUOH, N. U.; *et al.* Training based on virtual simulators: case study for improving parking skills. *In:* THE INTERNATIONAL SCIENTIFIC CONFERENCE ELEARNING AND SOFTWARE FOR EDUCATION. 2017, Bucharest: "Carol I" National Defence University, 2017. pp. 495.

NAGAI, Y.; TSUCHIYA, Y. Development of large-angle rear-wheel steering system perfecting small-turn performance and natural steering feeling at low velocity. *In:* 12th INTERNATIONAL MUNICH CHASSIS SYMPOSIUM 2021: CHASSIS. TECH PLUS. 2022, Heidelberg: Springer Vieweg, 2022. pp. 292-311.

NBC 5. No injuries in extreme parallel parking fail. **Car on top of car.** 03 nov. 2014. Available at: https://www.mynbc5.com/article/no-injuries-in-extreme-parallel-parking-fail/3320119. Accessed on: 22 Mar. 2023.

NEVES, V. D. O.; DELAMARO, M. E.; MASIERO, P. C. An environment to support structural testing of autonomous vehicles. *In:* 2014 BRAZILIAN SYMPOSIUM ON COMPUTING SYSTEMS ENGINEERING. 2014, Manaus: IEEE, 2014. pp. 19-24.

PAROMTCHIK, I. E; LAUGIER, C. Autonomous parallel parking of a nonholonomic vehicle. *In:* PROCEEDINGS OF CONFERENCE ON INTELLIGENT VEHICLES. 1996, Tokyo: IEEE, 1996. pp. 13-18.

PARK, Y.; AHN, J.; PARK, J. Deep learning based parking slot detection and tracking: PSDT-Net. *In:* ROBOT INTELLIGENCE TECHNOLOGY AND APPLICATIONS 6: RESULTS FROM THE 9TH INTERNATIONAL CONFERENCE ON ROBOT INTELLIGENCE TECHNOLOGY AND APPLICATIONS. 2022, Cham: Springer International Publishing, 2022. pp. 291-302.

PAUWELS, K.; *et al*. New products, sales promotions, and firm value: the case of the automobile industry. **Journal of Marketing**, Newbury Park, v. 68, n. 4, pp. 142-156, oct. 2004.

PETROV, P.; NASHASHIBI, F. Automatic vehicle perpendicular parking design using saturated control. *In:* 2015 IEEE JORDAN CONFERENCE ON APPLIED ELECTRICAL ENGINEERING AND COMPUTING TECHNOLOGIES (AEECT). 2015, Amman: IEEE, 2015. pp. 1-6.

RAZINKOVA, A.; CHO, H. C.; JEON, H. T. An intelligent auto parking system for vehicles. **International Journal of Fuzzy Logic and Intelligent Systems**, Daejeon, v. 12, n. 3, pp. 226-231, sep. 2012.

RAJA, A. S. R. M.; *et al.* Model-Based Energy Path Analysis of Tip-In Event in a 2WD Vehicle with Range-Extender Electric Powertrain Architecture. **Energies**, Basel, v. 14, n. 18, pp. 5696, sep. 2021.

REITZ, R. D.; *et al.* IJER editorial: the future of the internal combustion engine. **International Journal of Engine Research**, Newbury Park, v. 21, n. 1, pp. 3-10, jan, 2020.

RIVA, F. **Mapping-based improvements in autonomous perpendicular parking based on a low resolution lidar.** 2020. Master Thesis - School of Industrial and Information Engineering, Polytechnic University of Milan, 2020.

ROYSTON, A. **Henry Ford and the assembly line**. New York: The Rosen Publishing Group, Inc, 2015.

SAMUEL, M. Performance and robustness analysis of model predictive controller (MPC) and proportional integral derivative control (PID) of an autonomous vehicle for lane keeping maneuvers. **Preprints**, Basel, v. 1, dec. 2019.

SCHIEBEN, A.; *et al.* Designing the interaction of automated vehicles with other traffic participants: design considerations based on human needs and expectations. **Cognition, Technology and Work**, London, v. 21, n. 1, pp. 69-85, sep. 2019.

SEDIGHI, S.; NGUYEN, D. V.; KUHNERT, K. D. Implementation of a parking state machine on vision-based auto parking systems for perpendicular parking scenarios. *In:* 2019 6th INTERNATIONAL CONFERENCE ON CONTROL, DECISION AND INFORMATION TECHNOLOGIES (CODIT). 2019, Paris: IEEE, 2019. pp. 1711-1716.

TAN, X.; LIU, D.; XIONG, H. Optimal control method of path tracking for four-wheel steering vehicles. **Actuators**, Basel, v. 11, n. 2, pp. 61, feb. 2022.

TESLA. Eight cameras and powerful vision processing provide 360 degrees of visibility at up to 250 meters of range. **Advanced sensor coverage.** 18 nov. 2016. Available at: https://www.tesla.com/autopilot. Accessed on: 15 Mar. 2023.

TESLA. Dual motor all-wheel drive. **Model S.** 18 nov. 2016. Available at: https://www.tesla.com/models/design#overview. Accessed on: 15 Mar. 2023.

VIEIRA, R. P.; ARGENTO, E. V.; REVOREDO, T. C. An autonomous parallel parking algorithm for car-like mobile robots. **Journal of Control, Automation and Electrical Systems**, Campinas, v.33, n.6, pp. 1762–1772, jun. 2022.

VISWANATH, P.; *et al.* End to end learning based self-driving using JacintoNet. *In:* 2018 IEEE 8th INTERNATIONAL CONFERENCE ON CONSUMER ELECTRONICS-BERLIN (ICCE-BERLIN). 2018, Berlin: IEEE, 2018. pp. 1-4.

VISWANATH, P.; *et al*. Virtual simulation platforms for automated driving: key care-about and usage model. **Electronic Imaging**, Springfield, v. 2018, n. 17, pp. 164-1-164-6, nov. 2018.

VOROBIEVA, H.; *et al.* Automatic parallel parking in tiny spots: path planning and control. **IEEE Transactions on Intelligent Transportation Systems**, Blacksburg, v. 16, n. 1, pp. 396-410, feb. 2015.

WALMSLEY, M. R. W.; *et al.* Carbon emissions pinch analysis for emissions reductions in the New Zealand transport sector through to 2050. **Energy**, Amsterdam, v. 92, n. 3, pp. 569-576, dec. 2015.

WOLF, D. F.; *et al.* Intelligent robotics: from simulation to real world applications. *In:* MINI-COURSE: DAY OF UPDATING IN INFORMATICS (JAI), SBC CONGRESS. 2009, São Carlos: SBC Computing Update Conference, 2009. pp. 13.

YIN, Y.; STECKE, K. E.; LI, D. The evolution of production systems from Industry 2.0 through Industry 4.0. **International Journal of Production Research**, London, v. 56, n. 1-2, pp. 848-861, nov. 2018.

YU, A.; *et al.* Life cycle environmental impacts and carbon emissions: a case study of electric and gasoline vehicles in China. **Transportation Research Part D: Transport and Environment**, Amsterdam, v. 65, pp. 409-420, dec. 2018.

ZHELTIKOV, A. M. Information geometry, pythagorean-theorem extension, and euclidean distance behind optical sensing via spectral analysis. **Laser Physics Letters**, Bristol, v. 19, n. 6, pp. 065401, may. 2022.

APPENDIX A - Curriculum summary

ACADEMIC TRAJECTORY DURING THE DOCTORAL COURSE

During the doctoral course, ideas and concepts corresponding the maneuvers assistance algorithms were validated through scientific publications, where; the first article was focused on the kinematic and dynamic modeling of a four-wheeled vehicle.

The second article described the geometric aspects for the best performance in parking maneuvers; the third article validates the simulation of a predictive controller applied to a vehicle steering system.

The fourth article set the best PID controller tune method for vehicle plant control; and the fifth article describes the longitudinal modeling of a four-wheeled vehicle.

Therefore, for more consistency, credibility, robustness, and bibliographic update of my doctoral thesis, I highlight my articles published in international and national journals and congresses that provide high-level information’s for the development and validation of the driver assistance to park the vehicle automatically without human effort:

Paper 1: Mathematical modeling attributed to kinematics and dynamics of a vehicle with 4-wheels. *The European Physical Journal Special Topics.* 2021.

Author: Manuel, Calequela JT.; Santos, Max MD and Tusset, Angelo M.

Abstract:

This paper describes the mathematical modeling corresponding to the kinematics and dynamics of a vehicle with 4-wheels (light vehicles), to determine that the vehicle design meets and guarantees better driveability on a local road or highway. The mathematical modeling referring to the kinematics and dynamics (factors influencing vehicle performance) is proposed for traction on the front 2-wheels and for the 4-wheels of the vehicle, allowing to classify the type of vehicle and where it can be implemented. Is presented and justified the computational simulation of kinematic variable that directly influences vehicle dynamics.

Keywords:

Mathematical modeling, Kinematics and dynamics, 4-Wheels vehicle, Steering vehicular, Ackerman geometry.

https://doi.org/10.1140/epjs/s11734-021-00238-2

JCR (2.891).

Paper 2: Geometric Aspects to Perform Maneuver in the Automatic Parking System. *ConBRepro*. 2021.

Author: MANUEL, Calequela João Tomé.; LENZI, Giane G.; SANTOS, Max MD and TUSSET, Angelo M.

Abstract:

This paper presents the planning and geometric path used to perform the automatic parking maneuver in parallel form, as well as the main variables used in the domain of maneuver constantly required in the algorithms of automatic parking of vehicles; in order to ensure greater security and less parking time.

Keywords:

Automatic parking, Pythagorean theorem, Geometric modeling, Ackerman steering.

https://aprepro.org.br/conbrepro/2021/anais/arquivos/09242021_060948_614d9ce450b68.pdf

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Paper 3: Predictive Control Applied to the Steering System of an Autonomous Vehicle. *Journal of Vibration Engineering and Technologies.* 2022.

Author: Manuel, Calequela JT.; Lenzi, Giane G.; Santos, Max MD and Tusset, Angelo M.

Abstract:

Purpose:

This paper proposes the computational modeling of a lateral and longitudinal action Model Predictive Control (MPC) controller to immediately control and correct the steering system of an autonomous/intelligent vehicle considered robust during its displacement.

Methods and Results:

The control is defined by the input variable δ (steering system) and projected by the MPC controller considering input and output restrictions, the mathematical model of the vehicle steering system and experimental parameters are set to validate vehicular steering control regarding its precision and safety. The configuration of the vehicle system block diagram and simulations were performed with the aid of Matlab/Simulink software.

Conclusions:

The results of the computational simulations are represented by the output variables (lateral position Y and yaw angle ψ) that positively validates the MPC to take over the steering and braking of a preferably autonomous vehicle in in situations of sudden longitudinal and lateral movements and in front of obstacles. The linear and discrete answers presented classify the MPC controller as robust.

Keywords:

Model Predictive Control, Autonomous Vehicle, Intelligent Vehicle, Non-linear Systems, Vehicular Systems.

https://doi.org/10.1007/s42417-022-00551-7

JCR (2.333).

Paper 4: Computational Validation of the Best Tuning Method for a Vehicle-Integrated PID Controller. *Modelling and Simulation in Engineering*. 2022.

Author: Manuel, Calequela JT.; Santos, Max.; Lenzi, Giane G and Tusset, Angelo M.

Abstract:

This paper validates and analyzes the robustness of the proportional-integral-derivative (PID) action controller from an open transfer function that integrates a proportional-integral (PI) action controller to obtain the response of a robust action control during the automatic parking maneuver of a vehicle where the simulations are based on 3 adjustment methods: Ziegler-Nichols (ZN), Chien-Hrones-Reswick (CHR), and Cohen-Coon (CC), and as a result of the computer simulations, it is determined the best performance index of the PID controller represented by mathematical and graphic equations with the help of MATLAB/Simulink software.

https://doi.org/10.1155/2022/3873639

Paper 5: Longitudinal Modeling of a Road Vehicle: 4-Wheel Traction. *International Journal of Robotics and Control Systems*. 2022.

Author: Manuel, Calequela JT.; Santos, Max MD.; Lenzi, Giane G and Tusset, Angelo M.

Abstract:

This paper presents the longitudinal modeling of a 4-wheel traction vehicle represented in a block diagram using Matlab®/Simulink® software. The proposed modeling is suitable to

be implemented in automatic parallel, oblique, or perpendicular parking systems considering speed cases between 5 km/h and 30 km/h. For the computational simulations, it was considered that the vehicle starts at rest and goes up a referenced or determined slope in degrees (°), with a sufficient rear reaction force to allow the vehicle to move until the engine produces sufficient torque. For the model of the tire variant, the magic formula (characterized by the sum of five vectors about an axis) was used. Three input signals were considered, slope, wind, and accelerator variation were considered in numerical simulations. The output signals are rear and normal front forces, vehicle speed, angular velocity, and engine acceleration. The longitudinal modeling proposed allows for easily reproducing the results and assigning new parameters to validate a Project, contributing positively both to the automotive industries and in innovation-based scientific research.

Keywords:

Computational modeling; 4-Wheel traction; Longitudinal dynamics; Calibration parameters; Vehicular system; Engine performance.

https://doi.org/10.31763/ijrcs.v2i2.698

The publication and acceptance of these papers confirmed the whole concept to leverage this thesis and bring relevant results.